# A STUDY OF DYNAMIC PERFORMANCE OF THE HYDRAULIC SERVOMECHANISM


By

NGO-SY-LOC





DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR


JANUARY, 1985

# A STUDY OF DYNAMIC PERFORMANCE OF THE HYDRAULIC SERVOMECHANISM

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
NGO-SY-LOC

to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

JANUARY, 1985

NGC-SY-LOC

A study of dynamic performance of the hydraulic servomechanism

January, 1985

[M.Tech, ME]   70p

   Supervisor: B. Sahay

2/1/85

CERTIFICATE

CERTIFIED that the thesis entitled; "A STUDY ON DYNAMIC PERFORMANCE OF THE HYDRAULIC SERVOMECHANISM", submitted by Mr. Ngo-Sy-Loc has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

IIT-Kanpur
January 1985

Birendra Sahay
Professor
Department of Mechanical Engineering
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

## ACKNOWLEDGEMENT

I express my gratitude to Dr. B. Sahay for his guidance and supervision throughout the course of this research project.

I wish to express my sincere appreciation of the assistance I obtained from Shri H.K. Nathani in preparation of this manuscript.

I am thankful to all staff members and officers of Mechanical Engineering Department, Indian Institute of Technology, Kanpur, whose names are not mentioned here but they will live forever in my heart.

IIT Kanpur<br>January 1985

\- Ngo-Sy-Loc

TABLE OF CONENTS

LIST OF FIGURES

## NOMENCLATURE

| | | |
|---|---|---|
| $A$ | Effective cylinder area | $cm^2$ |
| $d_c$ | Effective cylinder diameter | cm |
| $F$ | Initial opening area of spool valve | $cm^2$ |
| $d_v$ | Spool diameter | cm |
| $f$ | Opening area of balancing throtle valve | $cm^2$ |
| $C_d$ | Flow coefficient of spool valve | |
| $C_{d1}$ | Flow coefficient of throtle valve | |
| $X_o$ | Initial opening of spool valve | cm |
| $W$ | Area gradient of spool valve | cm |
| $K_1, K_2$ | Mechanical amplifying coefficient of walking lever | |
| $K_3$ | A constant, $0 < K_3 \leq 1$ | |
| $P$ | Pressure | $Kg/cm^2$ |
| $P_s$ | Supplied pressure | $Kg/cm^2$ |
| $P_{so}$ | Supplied pressure when load acting on the motor is absent | $Kg/cm^2$ |
| $P_1, P_2$ | Pressures in working motor chambers | $Kg/cm^2$ |
| $P_{10}, P_{20}$ | Pressures in working motor chambers in steady state | $Kg/cm^2$ |
| $\Delta P_1, \Delta P_2$ | Changes of pressures over the steady state values | $Kg/cm^2$ |
| $Y_1, Y_2$ | Components of the controlled output | cm |
| $Y, \Delta Y$ | Position of driven element and its change | cm |
| $X, \Delta X$ | Input and its change | cm |
| $X_1, X_2, X_3$ | Components of motion of the spool | cm |

| | | |
|---|---|---|
| $V_c$ | Study velocity of driven element | cm/sec |
| $V_{max}$ | Maximum velocity of driven element | cm/sec |
| $\bar{v}$ | Non dimensional velocity | |
| M | Inertia load | $kg.sec^2/cm$ |
| $R, \Delta R$ | Load acting on driven element of the motor and its change | Kg |
| $\bar{R}$ | Non-dimensional load | |
| G | Weight of components | Kg |
| H | Volume of oil | $cm^2$ |
| $H_a$ | Volume of air mixed in the oil | $cm^3$ |
| $\varepsilon$ | Volume ratio | |
| e | Normalized input | |
| $e_o$ | half of dead zone | |
| $e^t$ | Dead zone | |
| $r, K_r, K_c$ | Constant coefficients | |
| $T_i, a_i$ | Constant coefficient of differential equation | |
| $B=\frac{\sqrt{2g}}{\sqrt{\gamma}}$ | A constant for a given fluid | $\frac{cm^2}{sec.kg^{1/2}}$ |
| g | Acceleration due to gravity | $cm/sec^2$ |
| $\gamma$ | Specific weight of oil | $Kg/cm^3$ |
| $\rho$ | Mass density of oil | $Kg\ sec^2/cm^4$ |
| $E=\frac{1}{\beta_e}$ | A constant for a given fluid | $cm^2/kg$ |
| $\beta_e, \beta_e'$ | Bulk modulus of oil without and with air mixed in it | $Kg/cm^2$ |

# ABSTRACT

It has been known that the phenomena governing the operation of hydraulic systems in general and hydraulic servosystems in particular are very complicated and can not be calculated theoretically with the desired degree of accuracy. Nevertheless, one is to admit that a theoretical study of a model of the hydraulic servosystems is very important, especially for the designers at designing and projecting stages to predict the characteristics of the designed systems.

Encouraged by this idea, to study transient response of a linear hydralic servomechanism, controlled by a two-edged underlap spool valve working with the inertia load has been chosen as the purpose of the present modest study.

The conventional method of modeling in hydraulic systems has been adopted to get static characteristics and the transfer function of the system. The transient response, then can be easily obtained by directly solving the characteristic equation, discribing the dynamic state of the system when a unit step input is given by D.E.C. using Runge-Kunta method.

The effects of the most important macro-geometrical parameters namely: diameter of the cylinder, initial opening area of the valve in the whole range of possible

pressures on transient response curves has been obtained.

The roles, played by various parameters such as : mechanical amplifying coefficient, value of inertia load and the percentage of air mixed in the working fluid have been estimated.

The obtained results could be used as a ground, for both users to predict dynamic performance of the servo-mechanism, and designers to select geometrical and working parmmeters of the system having pre-planed dynamic performance.

For completeness, the static characteric of the above mentioned type of servomechanism, have been obtained and presented in non-dimentional form.

A SHORT NOTE ON EARLIER WORK

Hydraulic servomechanism is an amplifying device which imparts to the driven element motion, co-ordinated with the rate, the direction and the finite degree of accuracy with that of the motion of the driving detecting elements.

The hydraulic servomechanism as a typical closed loop feedback control system has been known for quite sometime, but the science of servomechanism has gained prominence mostly during the last three decades only.

In the field of transportation (shipment, locomotive, heavy trucks and tractors) hydraulic servomechanisms are used as hydraulic-servo-steering systems. In mechanical engineering hydraulic servomechanisms are used to automatically machine components having given shape, which is sometime very complicated and of high degree of accuracy, which is very often beyond the physical abilities of man to duplicate. Here they are referred as Hydraulic-copy-heads or follow-up systems [9]. In automatic control engineering the most significant roles which hydraulic servomechanisms are playing are that of position control systems.

Hydraulic servomechanisms, due to their high dynamic performances, high energy ratio, longivity of service and compactness, are now being extensively used in chemical engineering, liquid processing systems, having and air

conditioning systems, in aircraft as well as space science. In short, at this stage, it is difficult to find out any branch of engineering where hydraulic servomechanisms are not being directly or indirectly used.

It is worth to mention that, although the fundamental research on flow phenomena in hydraulic valves has been carried out by S.Y. Lee [1] and the designs of different types of valves had been developed by William, S.E. [2] and latter by G.F. Kelly, John Bankers and M.E. Blanto [3], the theoretical characteristics of the typical dydraulic valves were determined by J.F. Blackburn [4], the idea of servomechanism was develoned by N. Minorsky [5] during the First World War period. It was N. Minorsky who moved the concept of a system which would automatically maintain a ship on a prescribed course. An error signal appeared in the result of difference between the desired course and the actual one, through an amplifying system will continuously act on the rudder of the ship so as to keep her on proper course. The contribution of N. Minorsky has since, laid a firm foundation for the development and utilization of servomechanisms in practice. Routh's [6] and Hurwitz's [7] stability criterions of a control system gave further impact to the development of theory of hydraulic servomechanisms. M. Guillon [8] had made an important advance in analysis and design in hydraulic servo systems widely used in aircract.

Very significance role in design and application in the field of machine tool has been made by E.M. Klimovich [9]. To date, the most completed work on systhesis and analysis of hydraulic servomechanisms is that of M.B. Tumarkin [10] where full clasification has been made and static characteristics of each type of system has been given.

It is not mistaken to say that all contributors to the theory of hydraulic servosystems used frequency response techniques to study theirdynamic performances. Very few [11] used analog computer as means to study dynamic performances of a given particular system. Bode [12] and Nyquist [13] have suggested more practical plotting methods of frequency response technique. Evan's [14] roots locus method had found its entrance in 1948 and soon became popular synthesis technique for single input-output linear servomechanism.

As shown in [10] linear hydraulic servomechanisms working on bridge-circuit principle (Fig. 1-a) can be classified into five groups and further each into nine subgroups depending on the design of the motors and the combinations of their control edges. A total number of the combinations reaches as much as 40. Each of them has different static characteristic and naturally dynamic performances. This is the reason, why a particular design which is widely used in practice should be chosen as a model to study.

## PRESENT WORK

Fig. 1-b depicts the design of the chosen system to be known as a linear hydraulic servomechanism controlled by two-edged under-lap four-way spool valve.

The advantages of the design is, when intial opening areas of the valve are large enough to allow whole flow-rate of the power unit to go through, the system can work with constant-pressure as well as constant flow-rate power units.

Chapter I is devoted to system description and its functional diagram.

In Chapter II, mathematic model, static characteristic and transfer function of the system have been developed.

Chapter III deals with system error, system stability as well as transient response analysis.

The obtained results describing the effect of the most important macro geometrical and working parameters, presented in graph form, along with their discussions, are given in Chapter IV.

In Chapter V, a theoretical comparison between transient curves of the system working in the constant flow rate and constant supplied pressure regimes has been given.

# CHAPTER I

# THE SYSTEM AND ITS SPECIAL FEATURES

## 1-1. SYSTEM DESCRIPTION

The system consists of a symmetrical linear motor (1), controlled by a two-edged under-lapped spool valve (2). The pressurised oil is supplied to the motor through two balancing throtle valves (9) and 10, by a power unit, consisting of a displacement pump (3), a filter (4), a pressure indicator (5), a relief valve (6), a radiator [7] and an oil tank (8). Assume that the piston (11) of the motor chambers will take care of inertia load, friction loads and external load acting on the motor body. The returning oil from the exit of the spool valve will follow the low-pressure tube (12) and back to the oil tank.

Input to the system is given in the form of the displacement of the walking lever (13).

## 1-2. WORKING OF THE SYSTEM AND ITS FUNCTIONAL DIAGRAM

In initial state, the spool takes its neutral position, just keeping balance of the bridge circuit in such a way that the driven element (in this case : body of the motor) is in the rest state.

Now, an input X is given as shown in Fig. 1-b, the spool will move to the RHS, decreasing the initial opning from the

right chamber side and increasing the one from the left chamber side. The balance of the bridge circuit is broken. As in the result, a different pressure across the motor chambers is created. When the different pressure is big enough to overcome the resistance of the loads, the driven element will move to the RHS. Since the driven element and the case of the spool valve are fixed together, the latter will also move to the RHS relative to the spool, just trying to return it back to its initial matural position. It is easy to notice that if the direction of the input X were changed to the LHS then the output Y would have moved to LHS too. If X is continuously given, Y also will have continous character. Similarly, different velocities of X will create different velocities of Y.

Just, by fixing together the cases of the motor and the spool valve, a special feature has been born to the system i.e., its driven element seems to have an ability to create a motion coordinated with the rate, the direction and the character of that of driving element.

Further to note that, while X is trying to increase the relative motion between the spool and its case, Y is, in contrary, trying to reduce it. Moreover, how big is the distant covered by the spool, exactly that big one will be covered by the case in the same direction, just when X stops acting after a while, the neutral position of the spool

valve is established. This phenomena, in control theory is known as a unit negative feedback which means the whole output Y is transferred back to the spool valve to compensate relative motion of the spool.

In order to draw functional diagram of the system, for the sake of clarity the motion of the system is imaginary considered as two simultaneous but separate motions namely:

1. The motion of the walking lever while the motor is at rest.
2. The motion of the motor while the lever is at rest.

In the first motion, when X is given, the spool will move a distant X1, which, according to Fig. 2-a, is defined as:

$$\frac{X1}{X} = \frac{BC}{AC} = K1$$

$$\therefore \qquad X1 = K_1 X \qquad (1\text{-}1)$$

The second motion can be divided into two sub-motions namely:

(a) motion of the motor while, point C on the lever is at rest.

(b) motion of point C while the motor is at rest. Considering the effect of Y on relative motion of the spool valve, the first sub-motion is nothing but the action of the negative feedback path due to which the case of the spool valve will move a distance

$$X2 = y \qquad (1-2)$$

In the sub-motion (b) due to the motion of point C, the spool will move a distant X3, which, according to Fig. 2-b is defined as

$$\frac{X3}{y} = \frac{AB}{AC} = K2$$

$$\therefore \quad X3 = K2.y \qquad (1-3)$$

In general case, the loads acting on the motor may not be constant. While the input X creates an output Y1, the fluctuation of the loads may create additional output Y2. Hence, the resultant output is defined as

$$Y = Y1 + Y2 \qquad (1-4)$$

The above discussion allows us to draw the functional diagram of the system as shown in Fig. 2-c.

The system is under two types of input : the given, prescribed input X and the distrurbance ΔR, whose the nature is unknown. In this study, our main concern is input X alone.

CHAPTER II

# MODELLING OF THE SYSTEM

## 2-1. MAIN ASSUMPTIONS

To describe the model of the system by mathematical language, the following assumptions were made:

- the motor-valve assembly is a perfect rigid construction under constant supply pressure $p_s$.
- no leakage between motor and valve chambers.
- no friction is acting on the driven element.
- there is no pressure loss in the connecting tubes, channels as well as the entrace and the exit of the motor.
- returning oil is under atmospheric pressure.

It is to note that, while the last assumption is reliable, the first two are reasonable in view of low operating pressure (20 $\div$ 40 Kg/cm$^2$) available in the system, the next two are not true in practice, nevertheless, for theoretical model, the above mentioned assumptions were usually accepted [9], [10] and recommended [12].

## 2-2. EQUATIONS DESCRIBING THE STATE OF THE SYSTEM

These equations are of three types, namely : the flow, the pressure and the motion ones. From Fig. 1-b, flow equation can be written as

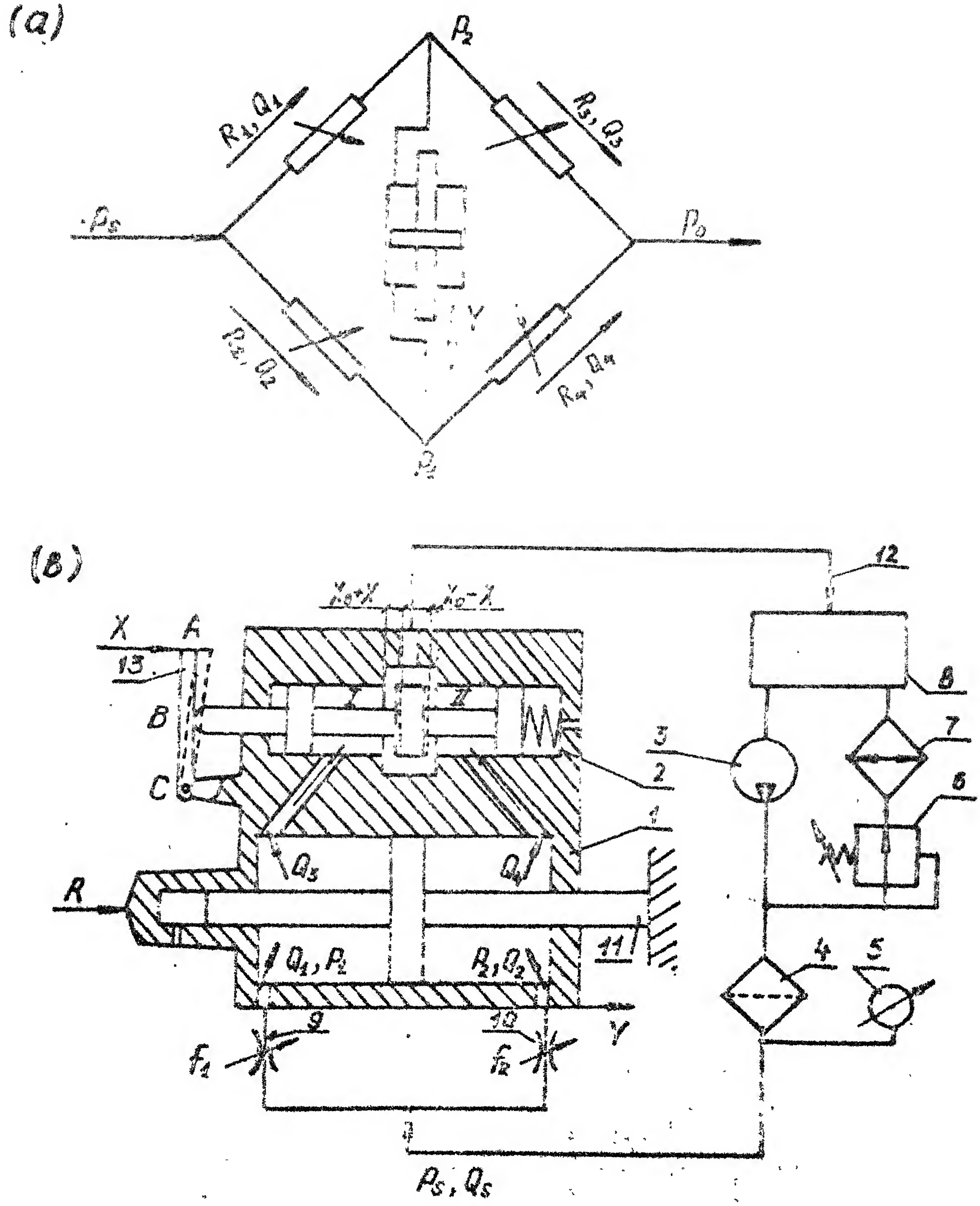


Fig. 1 Circuit scheme (a) and principle scheme (b) of the chosen hydraulic servomechanism

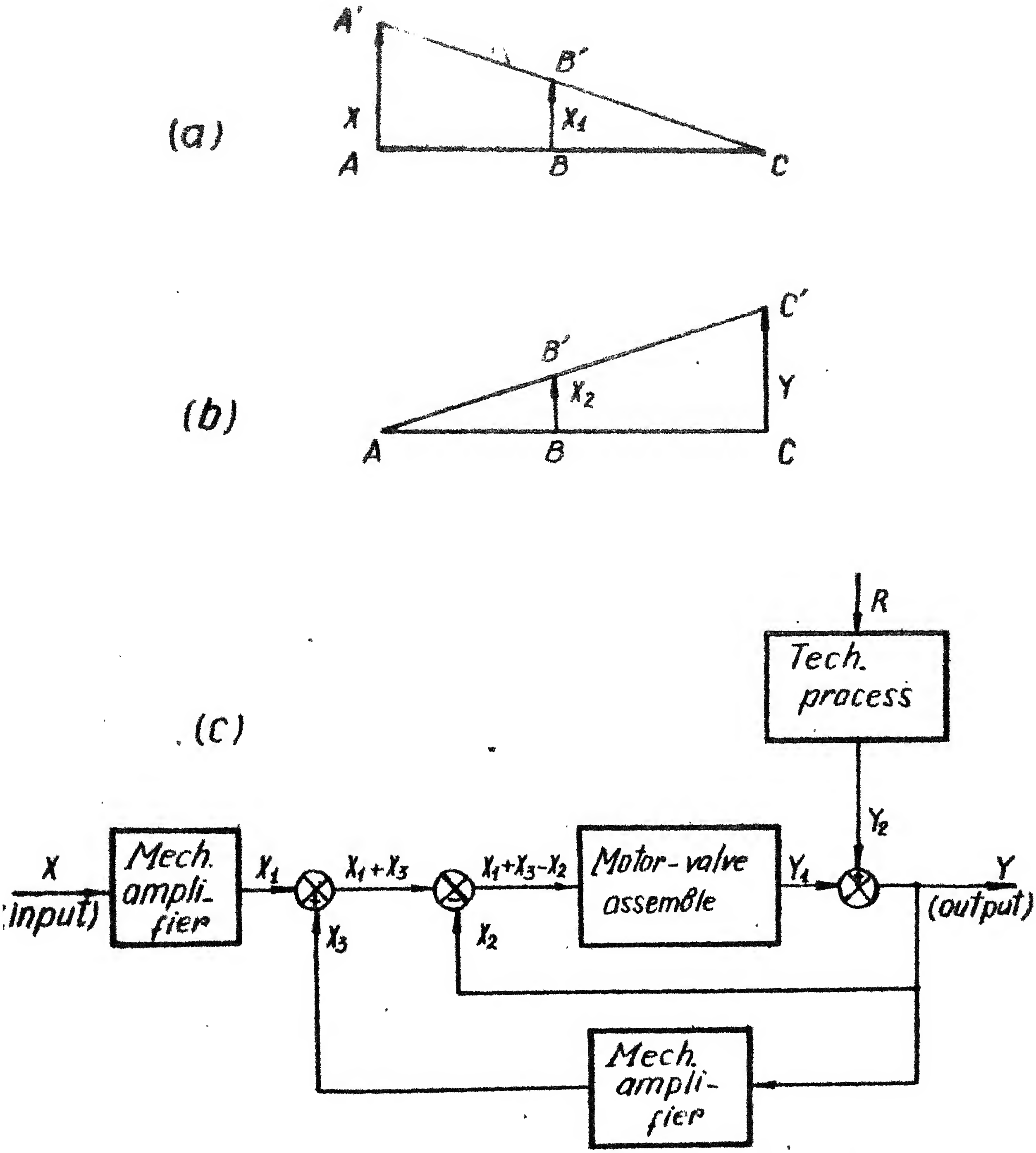


**Fig. 2 Functional diagram of the system.**

$$Q_2 = Q_4 + Q_6 + Q_8 \tag{2-1}$$

$$Q_1 = Q_3 + Q_5 - Q_7 \tag{2-2}$$

where
$$Q_1 = Cd_1 \cdot f_1 . F \sqrt{P_s - P_2} \tag{2-3}$$

$$Q_2 = Cd_2 \cdot f_2 . F . \sqrt{P_s - P_1} \tag{2-4}$$

$$Q_3 = Cd \cdot W \cdot B \cdot (x_{01} + x) \sqrt{P_1} \tag{2-5}$$

$$Q_4 = Cd \cdot W \cdot B \cdot (x_{02} - x) \sqrt{P_2} \tag{2-6}$$

$$Q_5 = E.V. \, d\, P_1/dt \tag{2-7}$$

$$Q_6 = E. \, V. \, dP_2/dt \tag{2-8}$$

$$Q_7 = Q_8 = A.dy/dt \tag{2-9}$$

For two-way motor with small load (R), we can assume that

$$f_1 = f_2 = f \tag{2-10}$$

$$x_{01} = x_{02} = x_0 \tag{2-11}$$

and
$$cd_1 = Cd_2 \tag{2-12}$$

Pressure equations are written as:

$$P_1 - P_2 = \frac{R}{A} \tag{2-13}$$

and
$$P_1 + P_2 = P_3 \tag{2-14}$$

Equation of motion of the dirven element CAN be

$$A(P_2 - P_1) + R = M \, d^2y \,/\, dt^2 \tag{2-15}$$

## 2-3. LINEARIZED EQUATIONS ESTABLISHED THE STATE OF THE SYSTEM

In view of non-linearity of the above equations, as mentioned in Sec. 1-3, Taylor's theory is used to get linearized equations. Resolve equations (2-1) - (2.9) and (2-15) into Taylor's series, remaining only linear quantities.

$$Q_{20}+ \Delta Q_2 = Q_{40}+ \Delta Q_4+Q_{60}+ \Delta Q_6+Q_{80}+ \Delta Q_8 \tag{2-16}$$

$$Q_{10}+ \Delta Q_1 = Q_{30}+ \Delta Q_3+Q_{50}+ \Delta Q_5-Q_{70}+ \Delta Q_7 \tag{2-17}$$

$$Q_{10}+ \Delta Q_1 = Cd_1.f.B. \sqrt{P_s-P_{10}} - \frac{Cd_1.f.B}{2\sqrt{P_s-P_{10}}} \Delta P1 \tag{2-18}$$

$$Q_{20}+ \Delta Q_2 = Cd_1.f.B. \sqrt{P_s - P_{20}} - \frac{Cd_1.f.B}{2\sqrt{P_s-P_{20}}} \Delta P2 \tag{2-19}$$

$$Q_{30}+ \Delta Q_3 = C_d.W.B.(x_o + \bar{x}) \sqrt{P_{10}}$$

$$+ C_d.W.B.\sqrt{P_{10}}. \Delta x + \frac{C_d.W.B.(x_o+\bar{x})}{2\sqrt{P_{10}}} \Delta P_1 \tag{2-20}$$

$$Q_{40}+ \Delta Q_4 = C_d.W.B.(x_o-\bar{x}) \sqrt{P_{20}}$$

$$+ C_d.W.B. \sqrt{P_{20}}. \Delta x + \frac{C_d.W.B.(x_o-\bar{x})}{2\sqrt{P_{20}}} \Delta P_2 \tag{2-21}$$

$$Q_{50}+ \Delta Q_5 = E.V.d \ \Delta P_1/dt \tag{2-22}$$

$$Q_{60}+ \Delta Q_6 = E.V.d \ \Delta P_2/dt \tag{2-23}$$

$$Q_{70}+ \Delta Q_7 = Q_{80}+ \Delta Q_8 = A.v_o+A.d\Delta y/dt \tag{2-24}$$

$$P_{20}+ \Delta P_2-P_{10}- \Delta P_1 + \frac{R_o}{A} + \frac{\Delta R}{A} = \frac{M}{A} . d \ \Delta y^2/dt^2 \tag{2-25}$$

Equations (2-13) and (2-14) accordingly became:

$$P_{10} + \Delta P_1 - P_{20} - \Delta P_2 = \frac{R_o}{A} + \frac{\Delta R}{A} \tag{2-26}$$

$$P_{10} + \Delta P_1 + P_{20} + \Delta P_2 = P_S \tag{2-27}$$

## 2-3. STATIC CHARACTERISTICS OF THE SYSTEM

Static characteristics of the system describe the state of the system in steady state, where the changes of all the parameters are to be zeros. Just, Equation (2-16) and (2-17) become:

$$Q_{20} = Q_{40} + Q_{60} + Q_{80} \tag{2-28}$$

$$Q_{10} = Q_{30} + Q_{50} - Q_{70} \tag{2-29}$$

Taking (2-18) ÷ (2-27) into account we have:

$$Q_{10} = C_{d1}.f.B.\sqrt{P_s - P_{10}} \tag{2-30}$$

$$Q_{20} = C_{d1}.f.B.\sqrt{P_s - P_{20}} \tag{2-31}$$

$$Q_{30} = C_d.W.B.(x_o + \overline{x})\sqrt{P_{10}} \tag{P-32}$$

$$Q_{40} = C_d.W.B.(x_o - \overline{x})\sqrt{P_{20}} \tag{2-33}$$

$$Q_{50} = Q_{60} = 0 \tag{2-34}$$

$$Q_{70} = Q_{80} = A.v_o \tag{2-35}$$

$$P_{10} - P_{20} = \frac{R_o}{A} \tag{2-36}$$

$$P_{10} + P_{20} = P_s \tag{2-37}$$

with these, the flow equations become:

$$Cd1.f.B.\sqrt{P_s - P_{20}} = Cd.W.B.(x_o + \overline{x})\sqrt{P_{20}} + Av_o \tag{2-38}$$

$$C_{d1}.f.B\ \sqrt{P_s - P_{10}} = C_d.W.B.(x_o - \overline{x})\sqrt{P_{10}} - Av_o \tag{2-39}$$

From (2-36) and (2-37)

$$P_{10} = \frac{P_s}{2} + \frac{R_o}{2A} \tag{2-40}$$

$$P_{20} = \frac{P_s}{2} - \frac{R_o}{2A} \tag{2-41}$$

From (2-38) ÷ (2-41) it is easy to see that

when load $R_o = 0$ and velocity $v_o = 0$

$$Q_{10} = Q_{20} = Q_{max} = C_{d1}.f.B.\ \frac{\sqrt{P_s}}{2} = C_d.W.B.x_o \frac{\sqrt{P_s}}{2} \quad (2\text{-}42)$$

The maximum possible velocity of the driven element is $v_{max}$ defined as:

$$v_{max} = \frac{Q_{max}}{A} \quad (2\text{-}42)$$

Let's denote

$$\frac{v_o}{v_{max}} = \bar{v} \quad (2\text{-}42)$$

$$\frac{R_o}{P_3 A} = \bar{R} \quad (2\text{-}43)$$

$$\frac{\bar{x}}{x_o} = e \quad (2\text{-}44)$$

Substitute (2-40) and (2-41) into (2-38) and (2-39), after dividing both sides of the both equations we gain:

$$\sqrt{1+\bar{R}} = (1-e)\ \sqrt{1-\bar{R}} + \bar{v} \quad (2\text{-}45)$$

$$\sqrt{1-\bar{R}} = (1+e)\ \sqrt{1+\bar{R}} - \bar{v} \quad (2\text{-}46)$$

from where non-dimensional velocity characteristic can be found as:

$$\bar{v} = \sqrt{1+\bar{R}} - \sqrt{1-\bar{R}} + \frac{e}{2}\ (\sqrt{1+\bar{R}} + \sqrt{1-\bar{R}}) \quad (2\text{-}47)$$

Velocity amplifying coefficient $K_v$ is defined as:

$$k_v = \left.\frac{\partial \bar{v}}{\partial e}\right|_{\bar{R}=\text{const}} = \frac{1}{2}(\sqrt{1+\bar{R}} + \sqrt{1-\bar{R}}) \tag{2-48}$$

Non-dimensional load characteristic can also be found from

$$\text{when } \bar{V} > 0; \ \bar{R} = \frac{\bar{v}^2}{e^2} - 1 \tag{2-49}$$

$$\text{and } \bar{V} < 0; \ \bar{R} = 1 - \frac{\bar{v}^2}{e^2} \tag{2-50}$$

Load amplifying coefficient can be found as

$$K_L = \left.\frac{d\bar{R}}{de}\right|_{\bar{v}=\text{const}} \tag{2-51}$$

From (2-49) :

$$K_L = \frac{2\bar{v}^2}{e^3}$$

Stiffness coefficient is defined as:

$$K_{LO} = K_L\Big|_{\bar{v}=0}$$

From (2-51), it appears that

$$K_{LO} = 0 \tag{2-52}$$

half of the dead zone can be found from (2-47) :

$$e_0 = e\Big|_{\bar{v}=0}$$

$$e_0 = 2\frac{\sqrt{1-\bar{R}} - \sqrt{1+\bar{R}}}{\sqrt{1-\bar{R}} + \sqrt{1+\bar{R}}} \tag{2-53a}$$

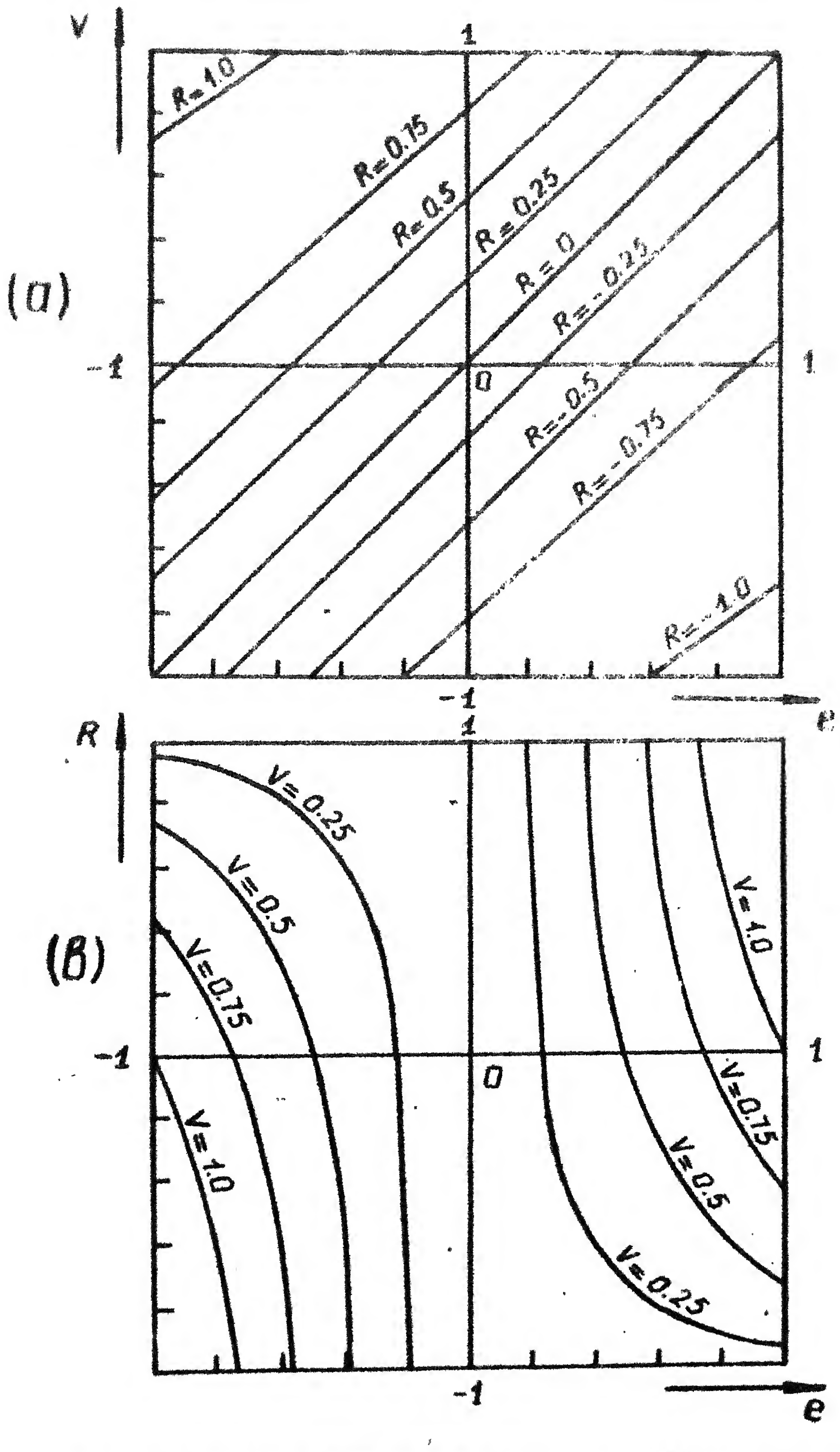


Fig.-3 Static characteristics of the system

In view of the symmetrical of the design the dead zone

$$e^* = 2e_c$$

$$e^* = 4\,\frac{\sqrt{1-R}-\sqrt{1+R}}{\sqrt{1-R}+\sqrt{1+R}}$$

Velocity ($V = V(e)/R = \text{const}$) and load ($R=R(e)/V=\text{const}$) Characteristics are shown on Fig. 3. It is clear that while $V(e)$ has linear character, $R(e)$ has non-linear character and discontinuity when $e = 0$.

A comparison based on static characteristics of different type of hydraulic servomechanisms is given by Turmarkin, M.B. [10].

## 2-4. TRANSFER FUNCTION AND BLOCK DIAGRAM OF THE SYSTEM

Flow equations in small changes of the parameters can be found from (2-16), (2-17), (2-28) and (2-29)

$$\Delta Q_2 = \Delta Q_4 + \Delta Q_6 + \Delta Q_8 \qquad (2\text{-}54)$$

$$\Delta Q_1 = \Delta Q_3 + \Delta Q_5 - \Delta Q_7 \qquad (2\text{-}55)$$

or

$$- C_{d1}.f.B.\ \frac{\Delta P_2}{2\sqrt{P_2-P_{20}}} = C_d.W.B.(x_o-\bar{x})\ \frac{\Delta P_2}{2\sqrt{P_{20}}}$$

$$- C_d.W.B.\sqrt{P_{20}}.\ \Delta x + A\ \frac{d\Delta y}{dt} + E.V.\ \frac{d\Delta P_2}{dt} \qquad (2\text{-}56)$$

$$- C_{d1}.f.B.\ \frac{\Delta P_1}{2\sqrt{P_s-P_{10}}} = C_d.W.B.(x_o-\bar{x}).\frac{\Delta P_1}{2\sqrt{P_{10}}}$$

$$+ C_d.W.B.\sqrt{P_{10}}.\ \Delta x - A.\ \frac{d\Delta Y}{dt} + E.V.\ \frac{d\Delta P_1}{dt} \qquad (2\text{-}57)$$

Equation of motion become:

$$\Delta P_2 - \Delta P_1 + \frac{\Delta R}{A} = \frac{M}{A} \cdot \frac{d^2 \Delta y}{dt^2} \qquad (2\text{-}58)$$

pressure equations become

$$\Delta P_1 + \Delta P_2 = 0 \qquad (2\text{-}59)$$

When small load is acting, in steady state, maximum flow-rate is defined as:

$$Q_{max} = C_{d1}.f.B. \frac{\sqrt{P_s}}{2} = C_d.W.B.x_o \frac{\sqrt{P_s}}{2} \qquad (2\text{-}60)$$

The following operations are to be per-formed:

Firstly: divide (2-56) and (2-57) by (2-60) and (2-58) by $P_{10} = P_{20} = P_s/2$. Then apply Laplace Transformation to the three just obtained equations, keeping in mind that the initial condition is zero one, finally eliminate the two intermediate parameters (pressure), after some simple rearrangements, the equation describing dynamic of the system can be written in the form:

$$(T_1.s^2 + T_2.s + 1)\ s.y = K_c x + (T_r.s + 1).K_r.f_r(s) \qquad (2\text{-}62a)$$

where

$$T_1 = \frac{E.M.V}{2.A^2}\ ;\ T_2\ \frac{M.B.(Cd.W.X_o + C_{d1}.f)}{2.\sqrt{2.P_s}.A^2};$$

$$T_r = \frac{\sqrt{2.P_s}.E.V}{B.(C_d.W.X_o+C_{d1}.f)} \quad ; \quad K_c = \frac{C_d.W.B.\sqrt{P_s}}{A.\sqrt{2}} \quad ; \qquad )2\text{-}62b)$$

$$K_r = \frac{E.\sqrt{P_s}.(C_d.W.X_o + C_{d1}.f)}{4.X_o.A.\sqrt{2}}; \quad f_r(s) = \frac{2.\Delta R(s)}{P_s.A} \quad ;$$

$$y = \frac{\Delta y}{X_o} \quad ; \quad X = \frac{\Delta x}{X_o};$$

In our case, taking (2-60) into account, the coefficients can be rewritten as:

$$T_1 = \frac{E.M.V}{2.A^2}; \quad T_2 = \frac{M.C_d.W.B.X_o}{A^2.\sqrt{2P_s}} \quad ;$$

$$T_r = \frac{E.V.\sqrt{P_s}}{\sqrt{2}.C_d.W.B.X_o} \quad ; \quad K_c = \frac{C_d.W.B.\sqrt{P_s}}{A.\sqrt{2}}; \qquad (2\text{-}62c)$$

$$K_r = \frac{C_d.W.B.\sqrt{P_s}}{2.A.\sqrt{2}} \quad ; \quad f_{r(s)} = \frac{2.\Delta R(s)}{P_s.A};$$

Rewrite (2-62a) into the following form:

$$y = \frac{K_c}{(T_1 s^2+T_2 s+1)s} X + \frac{(T_r s + 1)\ K_r}{(T_1 s^2 + T_2 s+1)s} . f_r(s) \qquad (2\text{-}63)$$

With the help of functional diagram (Fig. 2c) along with equation (2-63), the block diagram of the system can be drawn as shown in Fig. 4a. It is very clear that in general case, the system is subjected to two types of input. The first one is the prescribed given input : X and the second one is the disturbance, having random character, appeared in the result of fluctuation of load which the system is to overcome during its operation : $f_r(s)$.

Transfer function of the system with respect to the given input is defined whe N $f_r(s) = 0$,

$$W^x(s) = \frac{K_1 . K_c}{T_1 . s^3 + T_2 . s^2 + s + K_1 . K_c} \tag{2-64}$$

Similarly, transfer function of the system with respect to the disturbance due to load can be defined when $x = 0$; i.e.

$$W^r(s) = \frac{(T_r . s + 1) . K_r}{T_1 . s^3 + T_2 . s^2 + 1 + K_1 . K_c} \tag{2-65}$$

Fig. 4b shows the equivalent block diagram of that shown in Fig. 4a.

Fig. 4c shows the block diagram when only inertia load is acting on the driven element of the motor.

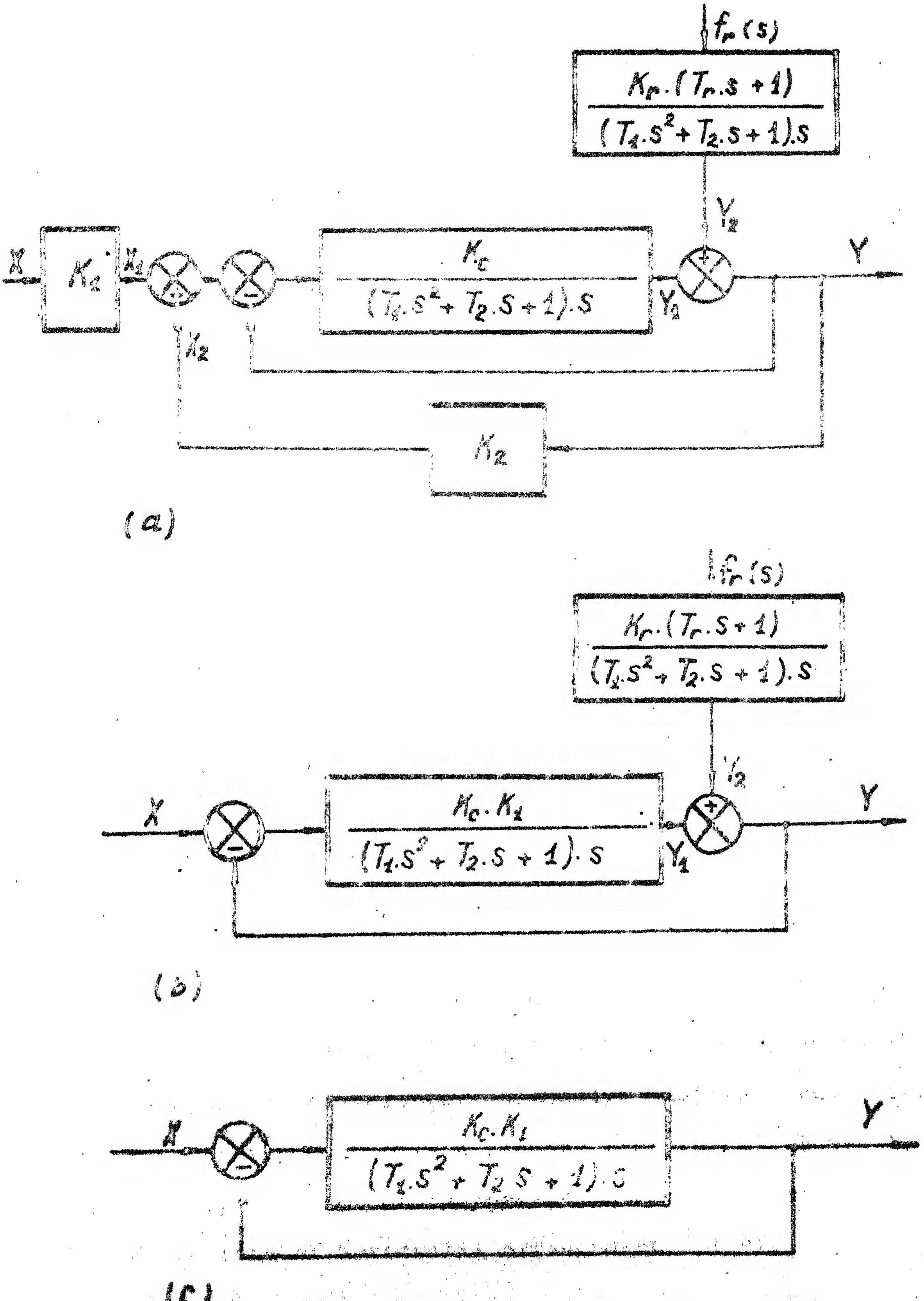


Fig. 4 Block diagram of the system.

# CHAPTER III

# SYSTEM ANALYSIS

## 3-1. ERROR ANALYSIS

If the motion is stable and the load acting on the driven element of the motor R is assumed to be constant then $dY/dt = \text{const.}$, $d^2y/dt^2 = d^3y/dt^3 = 0$ and we can find from Eqn. (2-64) the relative error due to velocity $h_v$.

$$h_v = \frac{dy/dt}{K_1.K_c} = \frac{A\sqrt{2}.\ dy/dt}{K_1.C_d.W.B.\sqrt{P_s}} \tag{3-1}$$

The error due to load, denoted by $h_r$ can be found from Eqn. (2.65), after putting $df_r/dt = d^2f_r/dt^2 = d^3f_r/dt^3 = 0$

$$h_r = \frac{K_r}{K_1\ .K_c} f_r(s) = \frac{\Delta R}{A\ .\ K_1.P_s} \tag{3-2}$$

Total error will be

$$h_a = h_v + h_r = \frac{A.\sqrt{2}.dy/dt}{K_1.C_d.W.B.\sqrt{P_s}} = \frac{\Delta R}{A.\ K_1.\ P_s} \tag{3-3}$$

It should be understood that load R in general, may consist of : firstly - friction in seals and slideways which is independent of velocity, secondly : cutting load which must be overcome by driven element (in case of copy-head) and lastly : weight of components moving in vertical direction (in case of horizontal arrangement G = 0).

From Eqn. (3-3) one can clearly see effect of the working parameters ($dy/dt$, $\Delta R$ and $P_s$) as well as geometrical parameters (the rest ones) on the value of error. While increasing some of parameters ($K_1$ and $P_s$) lead to the decreasing total error, increasing some other parameters ($C_d$, $W$ and $B$) can lead only to decreasing velocity erro r. On the other hand, velocity error is proportional to the area A but error due to load is proportional to the inverse of A.

Analysing Eqn. (3-3) many authors [9],[14] have suggested to take $K_1$ and $P_s$ big enough to achieve the desired error. But as shown in the next chapter, $K_1$, A and $P_s$ have very significance effects on the dynamic performances of the system. It is, therefore, suggested that when parameters such as: setting time, rise time, value of maximum overshoot .. are to be concerned as even second-order important in the designed system, the effects of the said parameters on transient curves should also be served as an additional ground along with the suggested Eqn. (3-3), at design stage.

From Eqn. (3-3), one can also see that, there is a value of A* which makes the total error minimum when other parameters are fixed. A* can be found from the condition : $dh_a/dA = 0$

$$A^* = \sqrt{\frac{C_d.W.B. \quad R}{\sqrt{2P_s}.dy/dt}} \tag{3-4}$$

3-2. STABILITY ANALYSIS

The purpose of any control system in general and of our hydraulic servomechanism in particular, is to 'keep' one or several parameters of the system in desired manner. For this purpose, the essential requirement to the system is: if any disturbance appeares in the system, it should disappear after sometime, or we can say that if an input is given, the system should be able to react in such a way that new stable state will be established after sometime. In theory of control, the system having the mentioned ability is called the stable system. A number of methods have been developed to enable us to check under what conditions the system is stable? If it is unstable then how to stabilize it?

In this section, Routh's criteria is used to test the absolute stability of the system. Routh's criteria is applied to polynomial having a finite number of terms and it tells us whether or not there are positive roots in the polynomial equation without actually solving for them.

As shown in Eqn. (2-64), the polynomial equation has the following form:

$$T_1 s^3 + T_2 s^2 + s + K_1 K_c = 0 \tag{3-5}$$

For Routh's analysis, the coefficient of the polynomial equation are to be placed and calculated [15] in Routh's table:

$$\begin{array}{ccc} T_1 & 1 & 0 \\ T_2 & K_1 . K_c & 0 \\ \dfrac{T_2 - T_1 . K_1 . K_c}{T_2} & 0 & 0 \\ K_1 . K_c & 0 & 0 \end{array}$$

For stability of the system, all coefficients in the first column should have the same sign, which is in the case, positive sign. Since parameters such as : $T_1, T_2$ and $K_1 . K_c$ are positive, it requires that

$$\frac{T_2 - T_1 . K_1 . K_c}{T_2} > 0 \tag{3-6}$$

from where $\qquad T_2 - T_1 . K_1 . K_c > 0 \qquad$ (3-7)

Substituting the values of $T_1, T_2, K_1$ and $K_2$ from Eqn. (2-62c) into Eqn. (3-7), after a simple operation, yields the following condition:

$$P_3 < \frac{2 . X_o}{K_1 . E . h} \tag{3-8}$$

Analysing Eqn. (3-8), it can be seen that the quanlity $2.X_o/h$ is a non-dimensional one and also $\acute{E}$ is a constant for a particular fluid- working in the system. The relation between working pressure ($P_s$) and the mechanical amplifying coefficient ($K_1$), for a particular nondimensional ratio is shown in Fig. 5. For $X_o$ = 0.005mm, h = 3.0 cm and modulus of the working fluid equal $1.4 \times 10^4 kg/cm^2$, the relation $P_s(K_1)$ is shown by the middle curve. Increasing as seen from the graph, the non-dimensional ratio leads to the shifting up of the curves. The pressure acceptable lies below the curve. The commercial under lap valve have their initial opening in the range $x_o$ = 0.002 to 0.006 cm [9], if the length of cylinder is 6cm, then their working pressure can be chosen in the range from 12 $kg/cm^2$ to 100 $kg/cm^2$ when $K_1$ varying from 5 to 0.8. For the middle curve, it is suggested to have $K_1$ between 0.5 to 2 then the working pressure will be from (20 to 70) $kg/cm^2$ which is sufficiently large for a system working with constant power unit.

As we assumed in steady state and in case only inertia load is acting $Cd_1.f.B\sqrt{P_s-P_1} = C_d.W.B.X_o\sqrt{P_1}$, which dictates that $P_1 = P_s/2$ and $C_d.W.X_o = Cd_1.f$, the latter means : the lower limit of the nondimensional ratio is half its value, is for the case when all other parameter are kept as they were but the initial opening of the value is reduced to 0.

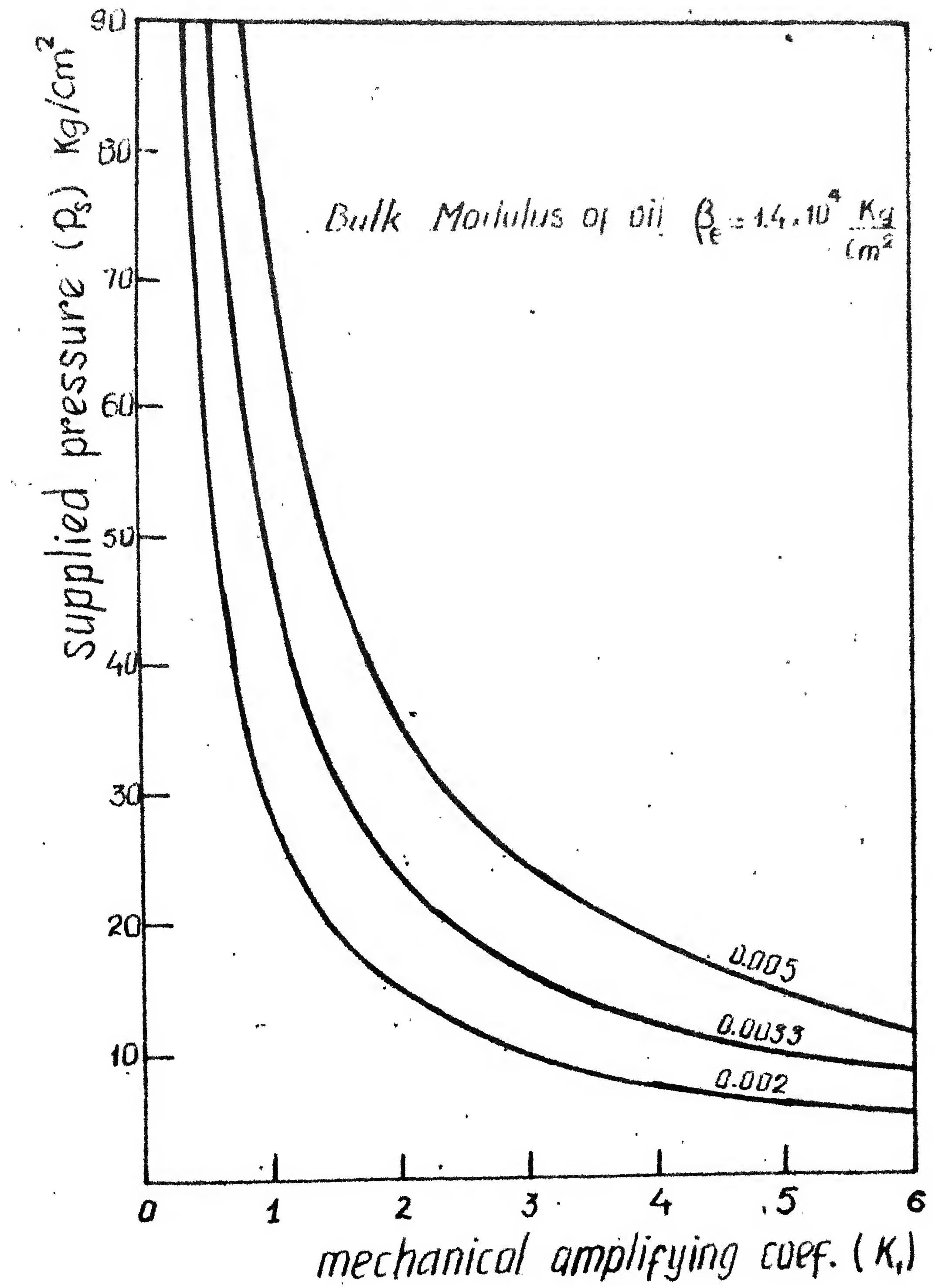


Fig-5 $P_s - K_1$ relation

Also from Eqn. (3-8), one can see that : since quality E is inverse of that of modulus of the working fluid, then, the fluid having high modulus should be chosen to work in the system, because this will give to the system better chance to be stable and higher upper limit of working pressure.

Summary

At this stage, system stability has been analysed but the nature of Routh's criteria does not show the degree of stability of the system. This is sometime very important in practice, for this, other methods namely : Root locus method and Bode diagram [15] are suggested to be employed. On the other hand, the system may be stable one but very often, quality of the transient response prevent it from utilization in practice. This dictates the need for analysing the transient response of the system even at design stage, so that a necessary correction could be made just to make the designed system having the desired dynamic performances.

## 3-3. TRANSIENT RESPONSE ANALYSIS

In view of the important role played by the quality of the transient response in operation of an automatic control system, this section is devoted to transient response analysis.

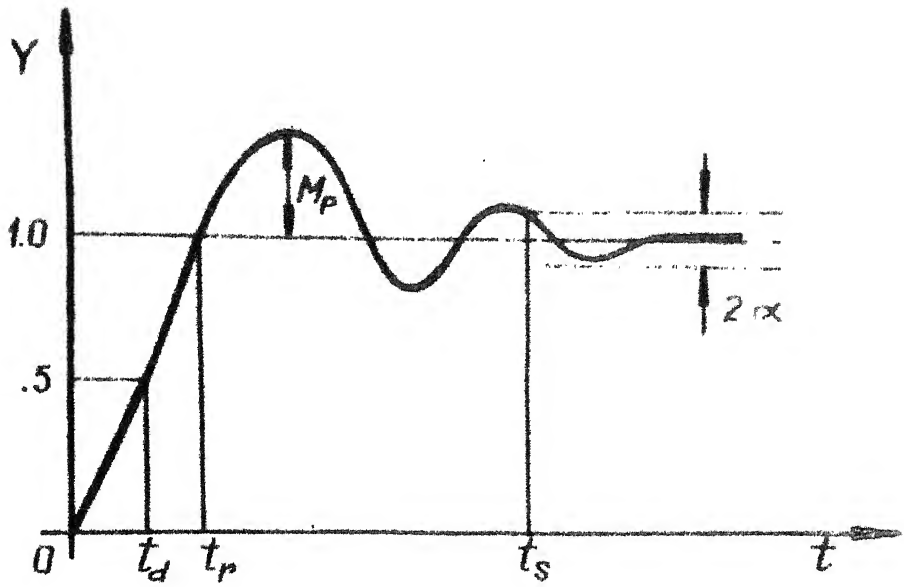


Fig. 6a - Typical transient curve

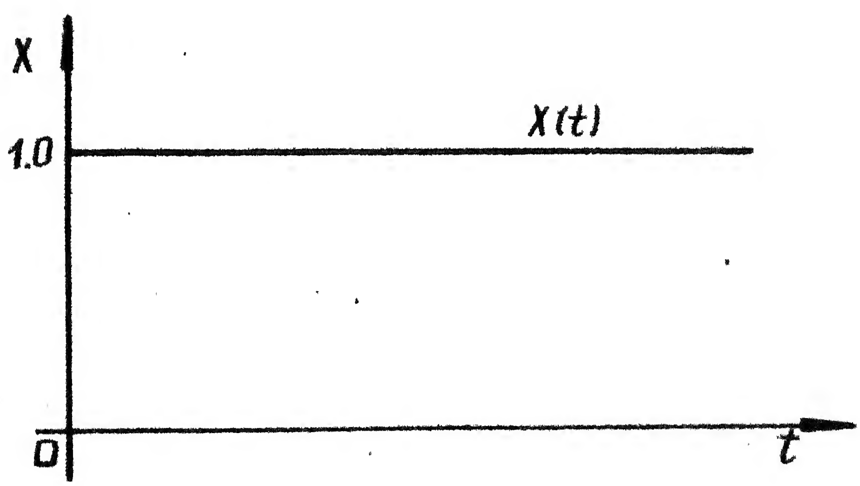


Fig. 6B - Typical step input

The quality of the transient response is characterized by the following quantities [15]:

(a) over shoot;

(b) delay time;

(c) rise time;

(d) settling time.

Fig. 6a shows graphic representation of the above mentioned quantities.

Other quantities are : Number of vibrations performed by the controlled parameter before it settles down, and the time at which the over shoot occurred.

For practical computation and comparison purpose, the most important parameters describing the quality of the transient response of the system would be : over shoot, delay time and settling time.

It was shown (Chapter II), the characteristic equation in terms of small changes of all the parameters, describing the dynamic state of the system can be rewritten in the form:

$$\frac{d^3y}{dt^3} + a_1 \frac{d^2y}{dt^2} + a_2 \frac{dy}{dt} + a_3 y = a_4 x \qquad (3\text{-}9)$$

where the constant coefficients are:

$$a_1 = \frac{4.B.(C_d.W.X_o + C_{d1}.f)}{h.\Pi.\ E.\ d^2.\sqrt{2.P_s}} \qquad (3\text{-}10)$$

$$a_2 = \frac{\Pi . d^2}{2.E.h.M} \qquad (3\text{-}11)$$

$$a_3 = \frac{K_1.C_d.W.B.\sqrt{2P_s}}{E.h.M} \qquad (3\text{-}12)$$

$$a_4 = a_3 \qquad (3\text{-}13)$$

Equation (3-9) is an ordinary linear differential equation; it's solution, when an unit step input is given, yields the complete expression for the performance of the system.

The unit step input shown in Fig. 6b, is characterised by the following expressions:

$$\begin{aligned} X(t) &= 0 \quad \text{when} \quad t < 0 \\ X(t) &= 1 \quad \text{when} \quad t = 0 \qquad (3\text{-}14) \\ \text{and} \quad X(t) &= 1 \quad \text{when} \quad t > 0 \end{aligned}$$

As shown in Eqns. (3-9) to (3-13), all geometrical as well as working parameters are in the expressions of the coefficients of the equation, this allows us to study the effects of the different parameters on the transient response of the system.

For a particular set of coefficients $a_i$, the Eqn. (3-9) was solved using the numerical technique, namely, Runge-Kutta method which, for this type of problem, had been generalized and had become a useful library subroutine [16]. To enable us to make use of the NAG programs, the Eqn. (3-9) is to be

written in the form of a system of the first order ordinary linear differential equation as follows:

Ler's first denote :

$$y_{(1)} = y$$
$$y_{(2)} = \frac{dy}{dt} \qquad (3\text{-}15)$$
$$y_{(3)} = \frac{d^2y}{dt^2}$$

further

$$F_{(1)} = \frac{dy}{dt} = y_{(2)}$$
$$F_{(2)} = \frac{dF_{(1)}}{dt} = \frac{d^2y}{dt^2} = y_{(3)} \qquad (3\text{-}16)$$
$$F_{(3)} = \frac{dF_{(2)}}{dt} = \frac{d^3y}{dt^3}$$

where, from Eqn. (3-9), $F_{(3)}$ can be written

$$F_{(3)} = - a_1.y(3) - a_2.y(2) - a_3.y(1) + a_4x(t) \qquad (3\text{-}17)$$

The initial conditions for the problem are as stated in Chapter II, i.e.,

$$y = \frac{dy}{dt} = \frac{d^2y}{dt^2} = 0 \text{ at } \quad t = 0 \qquad (3\text{-}18$$

The range of time is between 0 and t*. Where t* is the value of time when the transient response is terminated. t* is determined by a trial computation and then is to be treated as the chosen initial condition.

The obtained results have been presented in the graphical form and presented in the next chapter.

## CHAPTER IV

## RESULTS and DISCUSSION

### 4-1. EFFECT OF THE WORKING PARAMETERS ON TRANSIENT RESPONSE

For an under lap spool valve-motor combination, when all geometrical parameters and load are fixed, the only working parameter is supplied pressure $P_s$. Fig. 7 shows the transient curves of the system when pressure was changed from 10 $kg/cm^2$ to 80 $kg/cm^2$ with an increment of 10 $kg/cm^2$. The computation was made for the following available data : $d_c$= 6 cm; W= 5cm, $X_o$ = 0.005cm; h = 3 cm; M = 0.05 kg $sec^2/cm$; $K_1$= 0.5; $C_d$ = 0.7; $C_{d1}$ = 0.70; $\gamma = 875.10^{-6} kg/cm^3$; $\rho = 0.893.10^{-6}\ kg.sec^2/cm^4$; $\beta_e$ = $1.4x10^4\ kg/cm^2$ and f = $0.025 cm^2$.

If we take $t_s$, $M_p$ and $t_d$ as criteria to evaluate the quality of the transient response, then from the graph (Fig. 7) it is clear that -

- with the increasing $P_s$, $t_d$ decreases, which means the response of the system is faster.
- $t_s$, first decreases (upto $P_s$ = 50 $kg/cm^2$) and then increases due to vibration of the output caused by higher supplied pressure.
- overshoot occurs when $P_s \geq 50 kg/cm^2$, maximum value of overshoot is less than 15% which is acceptable for the most of practical control systems.

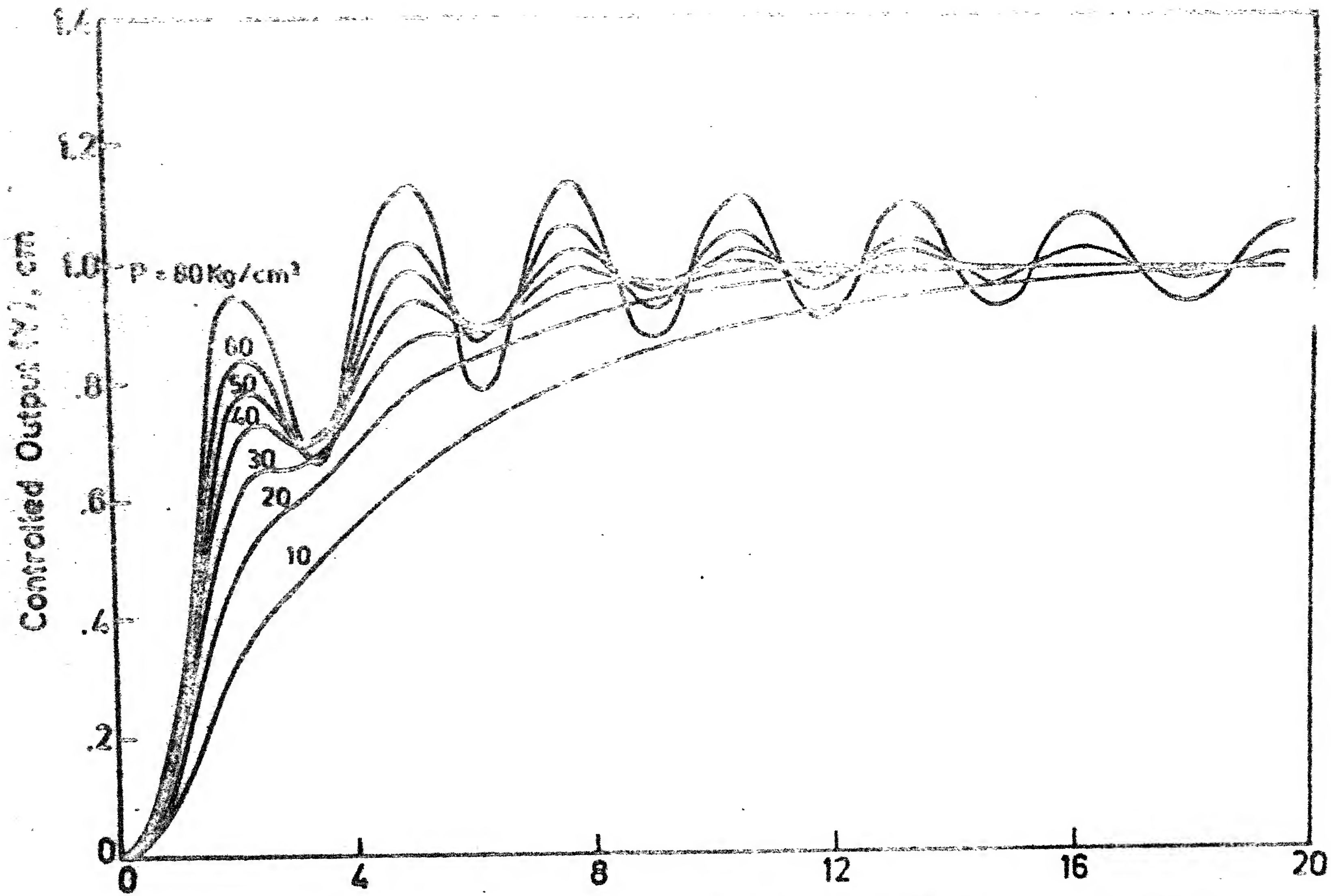


Fig. 7 Effect of pressure ($P_s$) on transient response

From the above discussion one can make a conclusion that for a particular design, there exists an optimum pressure where the system has its best settling time as well as acceptable overshoot and delay time.

## 4-2. EFFECT OF THE EFFECTIVE DIAMETER OF THE MOTOR ON TRANSIENT RESPONSE

When all parameters are fixed this means the supplied power to the system is fixed, the diameter of the motor will define the load that the system can overcome. An increment in $d_c$ will give to the system higher load capacity, but the actual load is constant, the inequality between the former and the latter will always affect on the transient response of the system.

Figure 8 shows the transient curves (Y) calculated under the same condition as adopted in Sec. 4-1 when supplied pressure is 30 $kg/cm^2$ and for different values of effective diameters : $d_c$= 3,4;6,8,10 cm.

It is clear that :

- increasing $d_c$ leads to decreasing the reaction of the system i.e. $t_d$ decdreases.
- when $d_c$ increase, $t_s$ first decdreases and then, after $d_c$ reaches some value ($d_c$= 4.2 cm in the case), $t_s$ has minimum value and this value is likely to be unchanged for the whole range $d_c$= (4.2 to 6) cm and lastly $t_s$ increases for $d_c > 6$ cm.

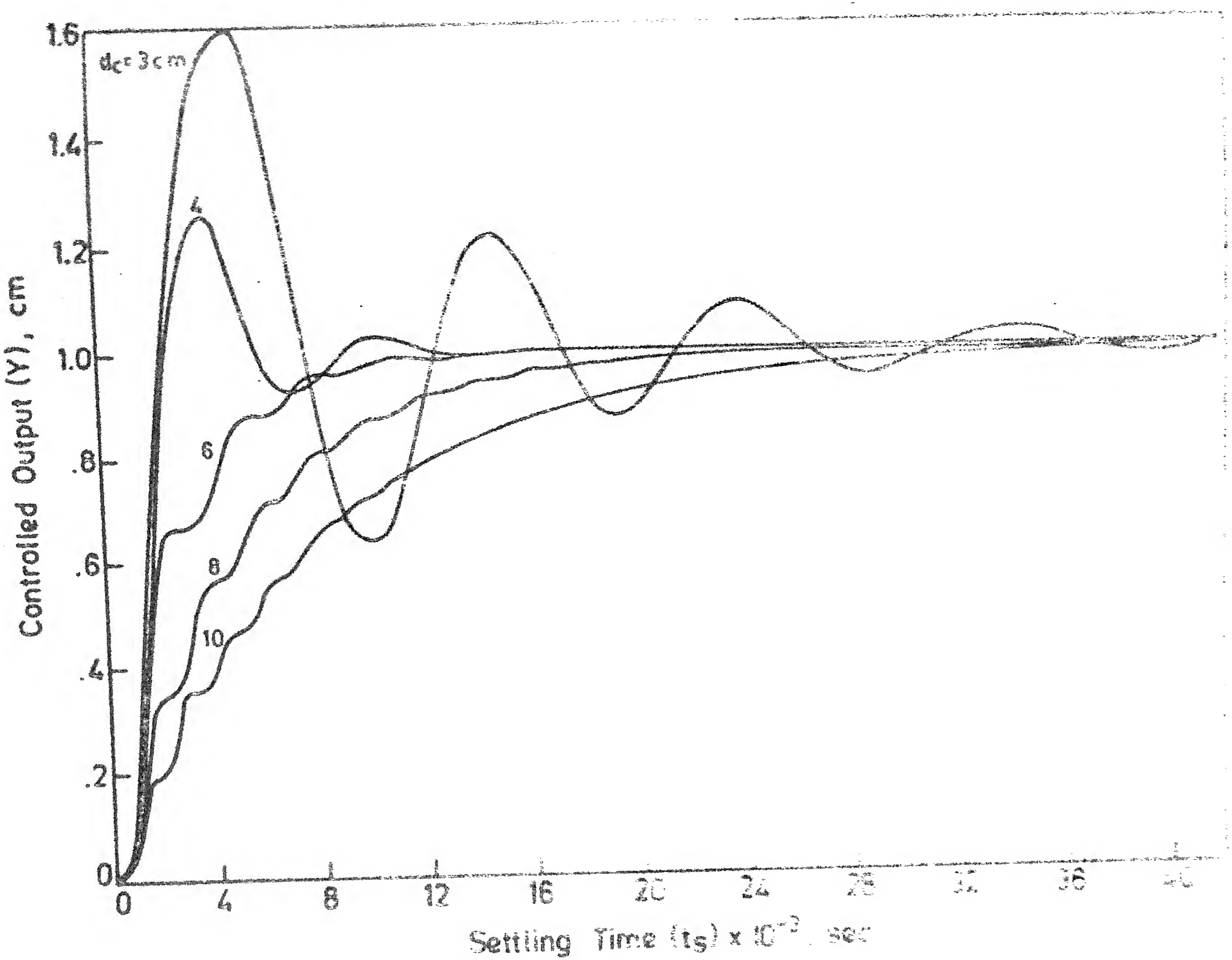


Fig 8 Effect of cylinder diameter ($d_c$) on transient response ($P_s$ = 30 Kg/cm$^2$)

- overshoot is zero when $d_c \geq$ 5cm and rapidly increases with decreasing value of $d_c$ ($d_c <$ 5cm)

These observations tell us that : there also exists an optimum value of $d_c$ where the system has its best settling time and at the same time it is acceptable values of overshoot as well as delay time.

From this conclusion and that of the previous section, it is very necessary to perform an operation of computation that can give us the relation between transient response and $d_c$ for each possible value of supplied pressure.

Such computation has been made and the obtained results are given in the form of the graphs shown in Figs. 9,10 and 11.

Figure 9 depicts the relation $t_s = f(d_c)$, for each $P_s$= const, it can be seen that in low pressure range (upto 30 kg/cm$^2$), the value of the effective diameter lies within (4.2 to 5.2) cm and at higher pressure the acceptable range of $d_c$ increases.

Figure 10 depicts the relation $M_p = f(d_c)$, for each $P_s$= const. One can see that for small diameter overshoot is very high, even at low pressure. There is a fall in overshoot at $P_s >$ 70 kg/cm$^2$. This happended because vibration of the controlled parameter due to high pressure. The bigger the diameters, the smaller the overshoot.

Figure 11 shows the relation $t_d = f(d_c)$ for each $P_s$= const. It turns out that when pressure increases, delay time decreases and also for each $P_s$= const., there is a range of $d_c$ where delay time is small. The said range is almost symmetrical around $d_c$= 4 cm and the width of the the zone is defined by the value of pressure as shown in the graph.

From Fig. 9, 10 and 11 we can draw the graph shown in Fig. 12. The inner area of ABB'A' is the zone of equal settling time for different motions working at the same pressure (with tolerance $\Delta t \leq 10^{-3}$ sec), the curve CDABC' is the border line, the area on the LHS of which is the zone of equal rise time for different motors working at the same pressure $P_s$. (with tolerance $\Delta t \leq 10^{-3}$sec) The time DD' is 10% overshoot line, the area on the RHS of DD' is the zone of acceptable overshoot (where overshoot is less than 10%) for different motors working at the same pressure. The common area dashed area, is the advisable zone, where different motors acting under the same supplied pressure will have the transient response which characterized by : the same $t_s$, $t_d$ (tolerance $\Delta t \leq 10^{-3}$ sec) and at the same time, their overshoot is less than 10%. For example when supplied pressure is equal 60 kg/cm$^2$ then the motors having different diameters between 5cm and 7 cm will have the same quality of transient response.

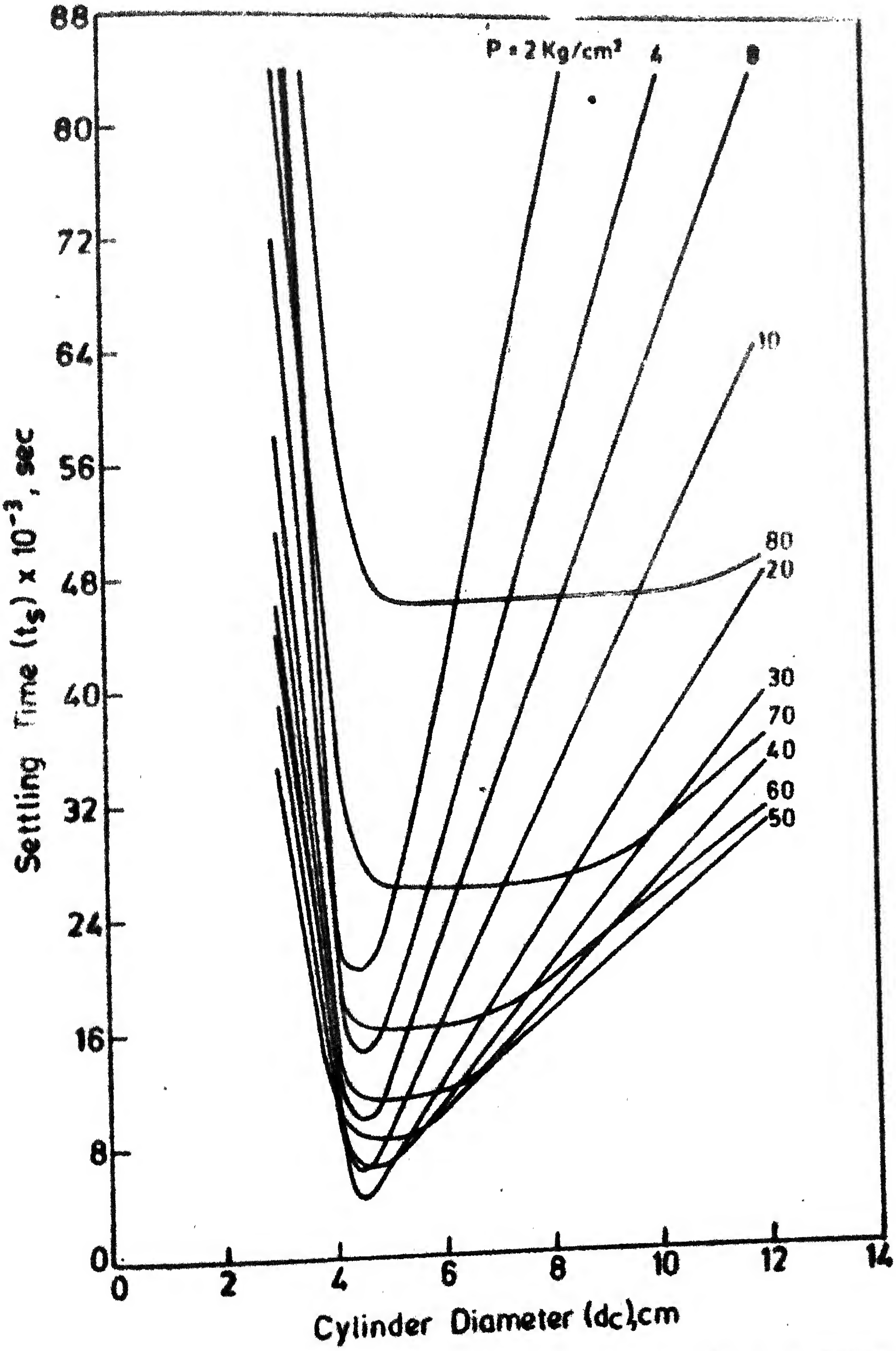


Fig. 9 Pressure ($P_S$) - cylinder diameter ($d_C$) - settling time ($t_S$) relation.

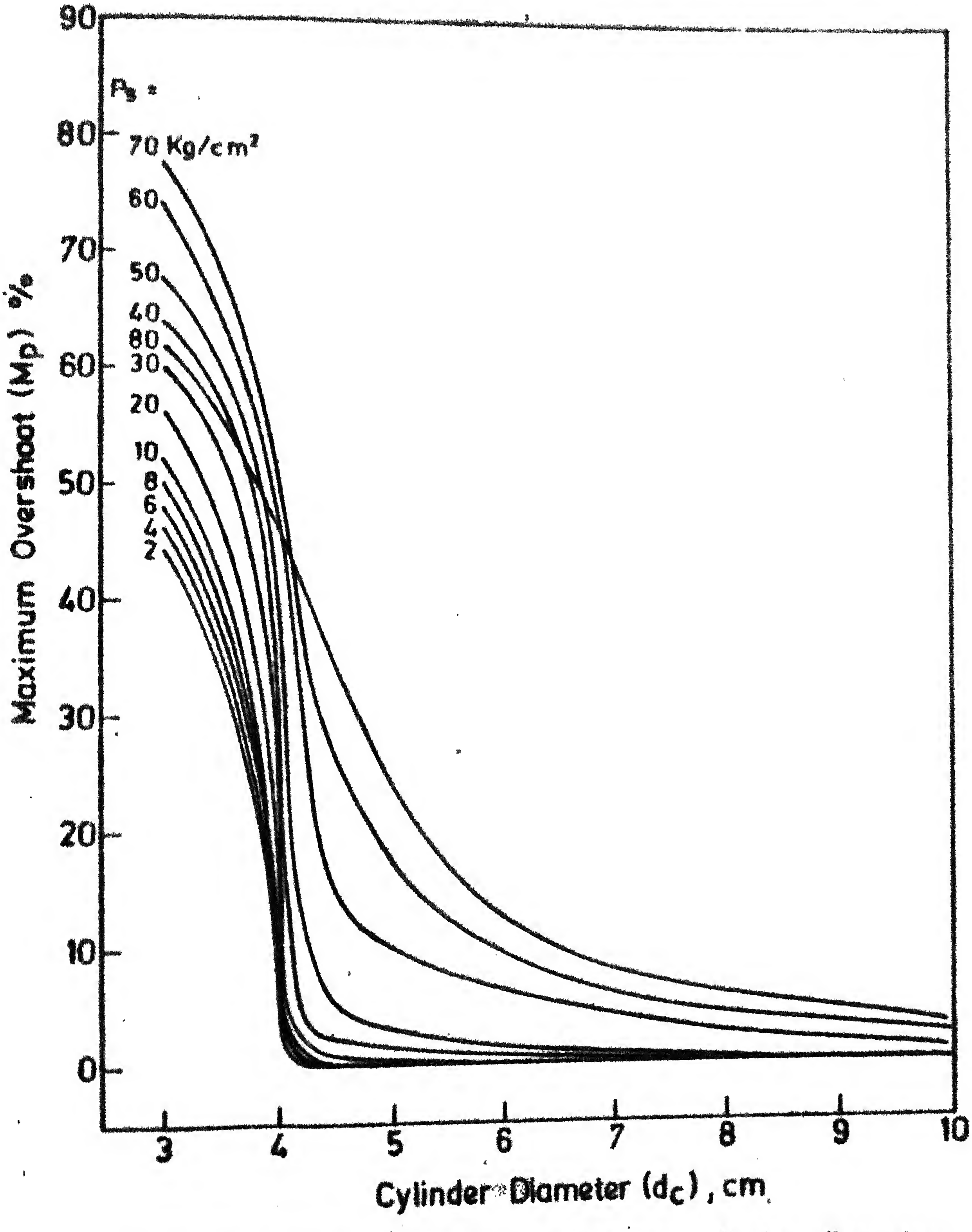


Fig. 10 Maximum overshoot - pressure - cylinder diameter relation.

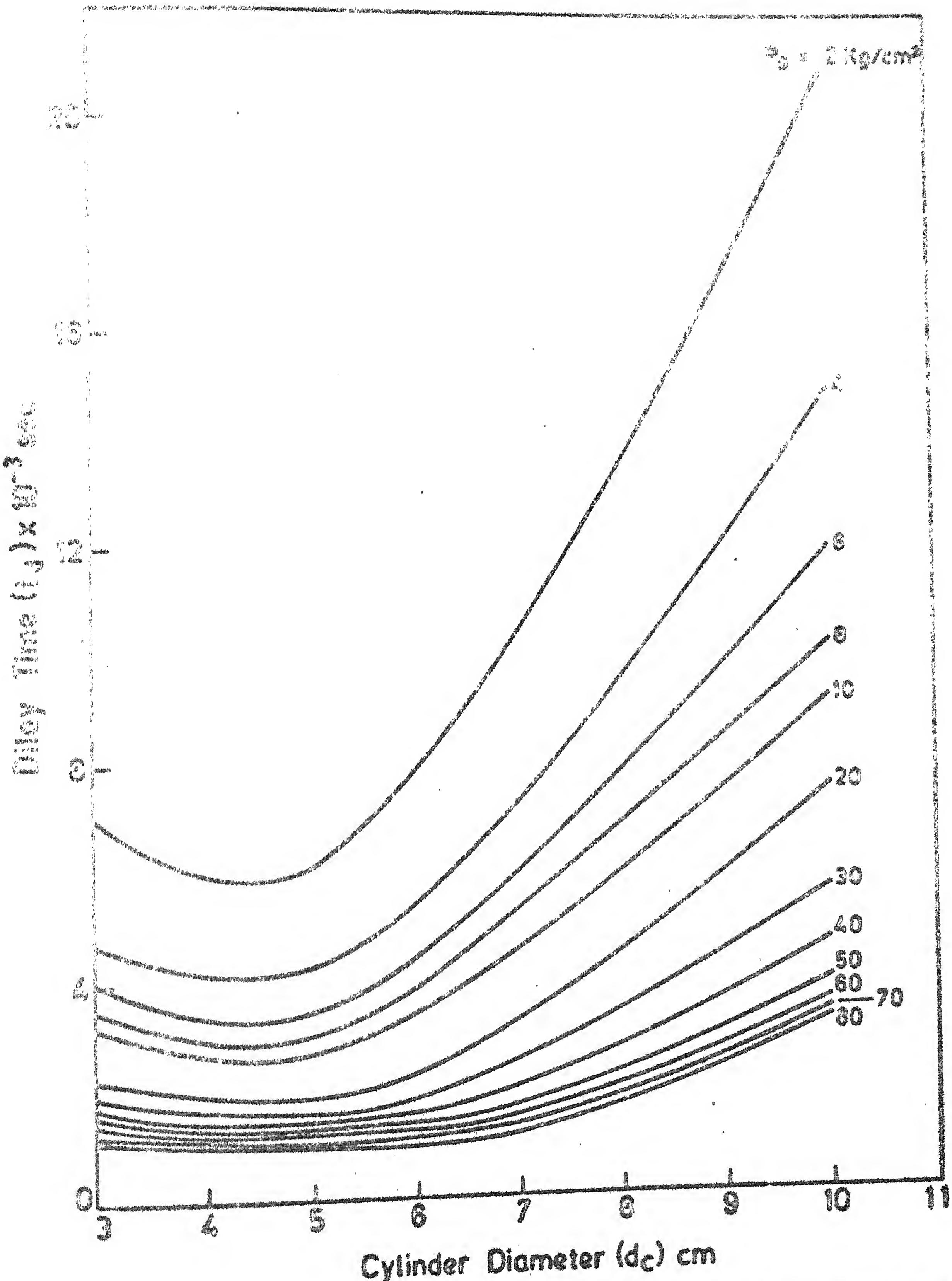


Fig. 11 Pressure ($P_s$) - cylinder diameter ($d_c$) - dilay time ($t_d$) relation.

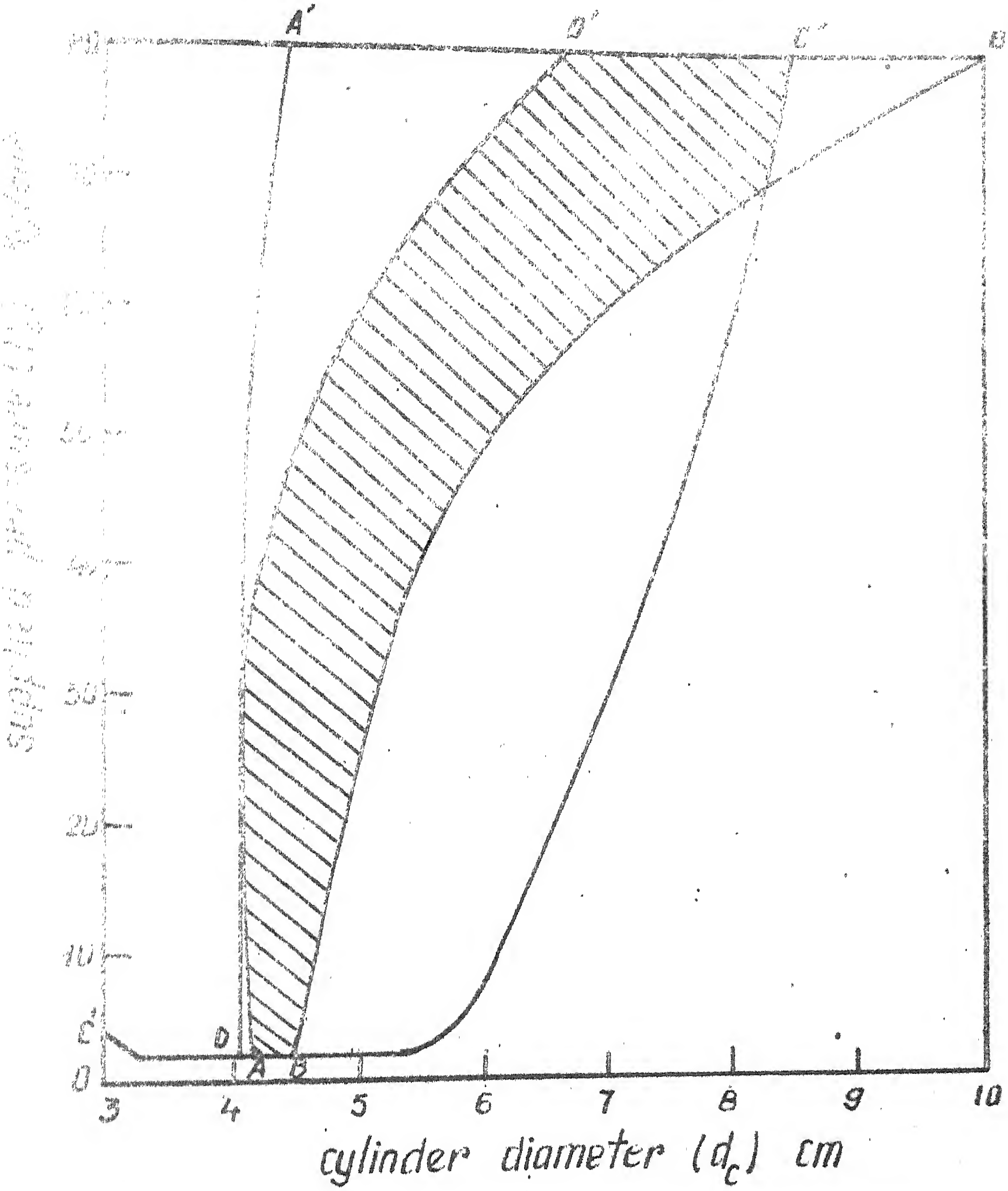


Fig. 12 Graph for selection diameter of cylinder $(d_c)$ w.t. $P_s$.

Of course, it should be noted that the other parameters should be the same, as adopted in Section 4-1.

## 4-3. EFFECT OF THE CROSS SECTIONAL AREA OF THE THROTLE VALVE ON THE TRANSIENT RESPONSE

As it was assumed in Section 3-2, the combination between the throtle valves and the spool valve is in such a way that the drop of pressure at each of the throtle valve is equal to half of that of the supplied pressure. This leads to the fact that $P_1 = P_2 = \frac{P_s}{2}$ in steady state and the cross sectional areas of the valves should be equal : $f = F = B.X_o = 5x0.005 = 0.025cm^2$.

Nevertheless, it will be interesting to know how f will affect the shape of the transient curves. Calculation with f = 0.015, 0.02, 0.03, 0.04 and $0.05cm^2$ had been made for the case K = 1.0, d = 6.5cm, P = $30kg/cm^2$ and the rest of the parameters were taken from Section 4-1. The results are shown in Fig. 13. As to our expectation, the cross-area of the throtle valve should be greater or at least equal to that of the spool valve then we will have 'smooth' shape of the transient curves. Decrease cross-area of the throtle valve will cause vibration of controlled output, this leads to increase in settling time from the graph, small cross-area of the throtle valve causes faster reaction of the system (td decreases) and greather overshoot. This seems to be a contradictory but a simple calculation shows that :

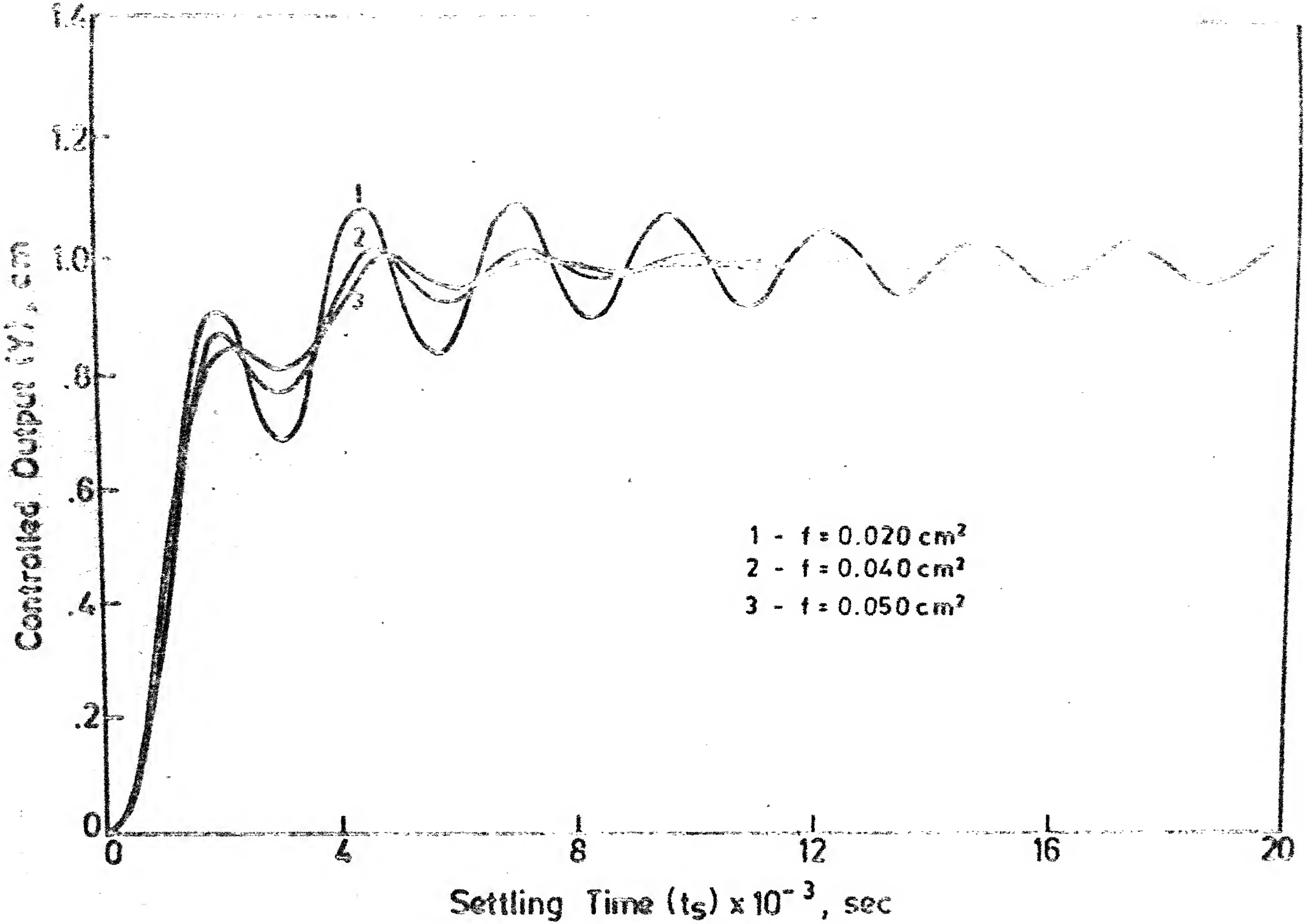


Fig. 13 Effect of opening area of balancing throtle valves on transient response.

when cross-area of the throtle valve is small, although pressures in the two cylinder chambers in steady state are smaller but when spool is opened, a greater pressure difference is created, the latter pushes the piston faster this causes decrease in delay time and increase in overhoot. But when piston moves, the supplied oil can not feel the chamber pressure fast enough, piston is forced to move backwards, this is why vibration of the controlled output occurs.

Calculation with the same set of cross areas in case $K_1 = 0.5$ had also been made but in this case, the shape of the transient curves does not change signficantly. This gives us a hint within which $K_1$ must play an important role.

## 4-4. EFFECT OF THE MECHANICAL AMPLIFYING COEFFICIENT $K_1$ ON TRANSIENT RESPONSE

The general effect of amplifying coefficient $K_1$ on transient response is clearly seen from the graph shown in Fig. 19 and Fig. 15., i.e. : an increament in $K_1$ leads to decreasing delay time of the system. While, at low pressure ($P_s = 20\ kg/cm^2$), by increasing $K_1$ we can achieve better quality of transient response in terms of settling time and value of overshoot. At higher pressure ($P_s \geq 30 kg/cm^2$) the quality worsens due to the appearance of overshoot, the latter leads to increase in settling time and even to

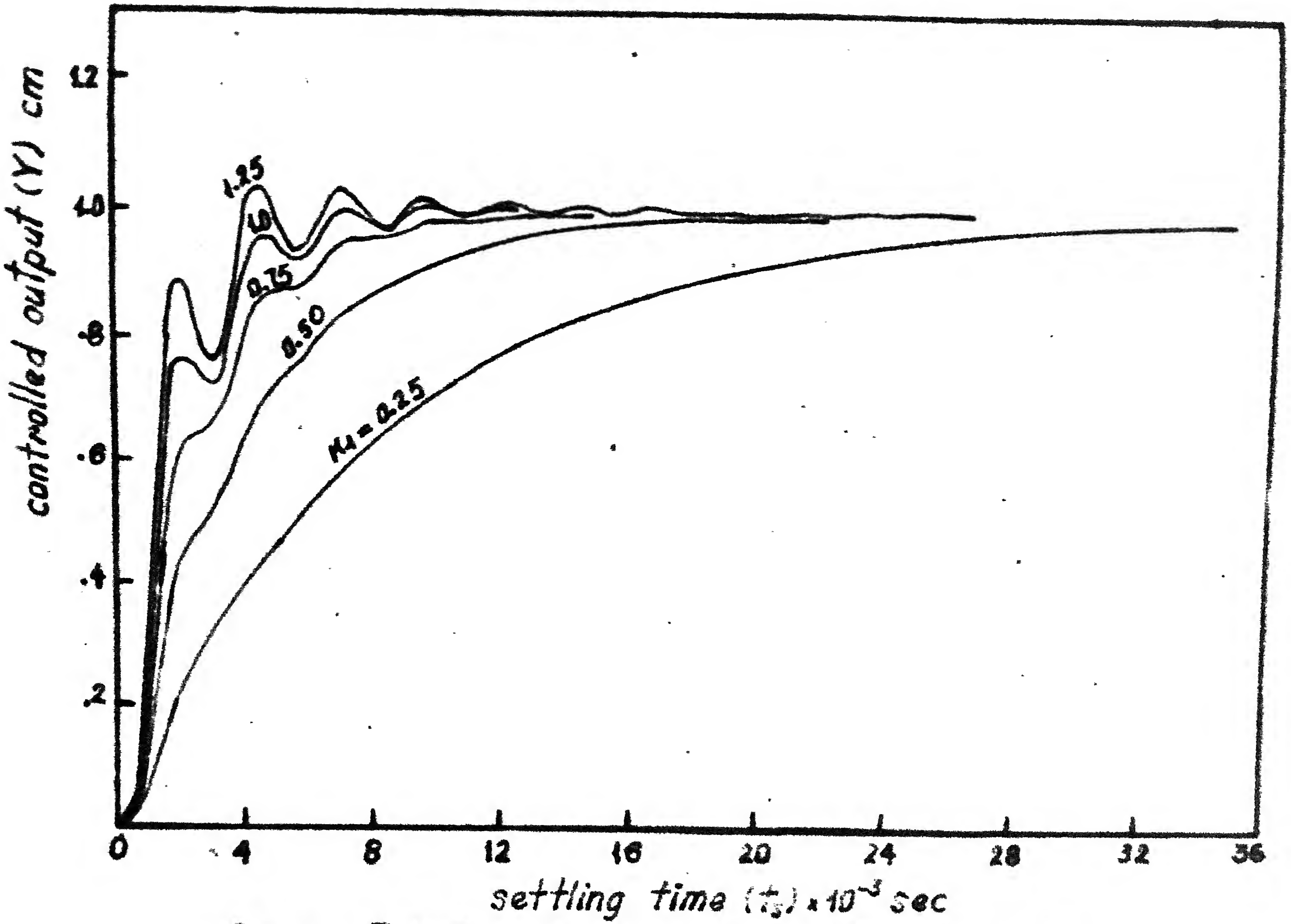


**Fig. 14** Effect of $K_1$ on transient response of the system ($P_s$ = 20 Kg/cm$^2$)

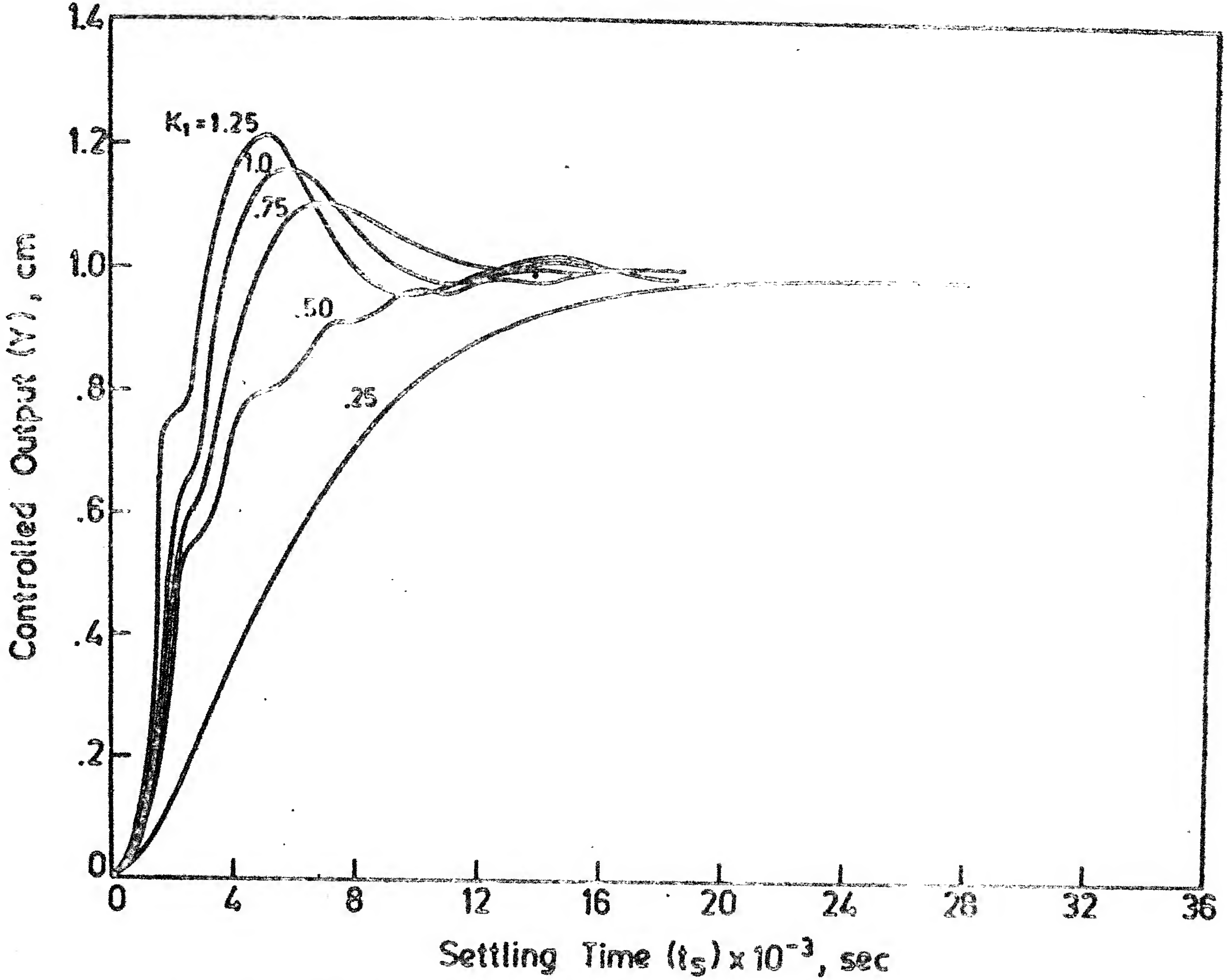


Fig. 15 Effect of $K_1$ on transient response ($P_s$ = 30 Kg/cm$^2$)

unstability of the system. The above observation is quite understandable if we return back to Fig. 5, which shows that at high pressure the range of $K_1$ is smaller than that at low pressure. Under given supplied pressure, when the question of selecting $K_1$ arises, Fig. 5 can be used for the purpose but taking the above discussion as well as the effect of pressure in to account it is suggested that some tolerance should be given to the curve shown in Fig. 5 (The curve itself is the border-line, on which the system will experience constant vibration, on LHS - the system will be in its stable state), and the tolerance for higher pressure should be greater than hat for lower pressure.

### 4-5. EFFECT OF INITIAL OPENING AREA OF THE SPOOL VALVE ON TRANSIENT RESPONSE

Initial opening area (F) of the spool valve is defined as product of area gradient (W) and initial opening ($X_c$). Here, $W = \Pi d_v K_3$; where $K_3$ is a constant. $0 < K_3 \leq 1$. In most of the practical cases $K_3 = 1$, which means the opening area is located around the spool. Industrial under-lap valves used to have the values of the initial opening from 0.002cm to 0.006cm [9]. The diameter of the spool, due to technical reason should be $d_v \geq 0.8$cm. This means, any standard under lap valves will have their opening area $F \geq 0.5 \times 10^{-2}$ cm$^2$, depending on the spool diameter. This dictates that a computation for the above range of F should be made to see its effect on the transient response

Figure 16 depicts the typical transient curves, obtained for a motor having $d_c$ = 4.5cm, working under supplied pressure $P_s$ = 30kg/cm$^2$. (Other parameters were taken from Section 4-1). It is clear that : an increament in F, leads to decreasing delay time, the reason for this is that: the higher the initial opening area, greater pressure difference across the piston will be created, and the piston will move faster. Contineous increase in F will cause the overshoot and vibration which leads to the increasein settling time.

The shape of transient response for the whole possible range of pressure is shown in Fig. 17, 18 and 19.

Figure 17 shows the relation $t_s = f(F)/P_s$ = const. The following observations can be made:

1. For each $P_s$ const, there exists a zone of F, where settling time has its best value, the zone is well located in between F = 2x10$^{-2}$ cm$^2$ and F = 3x10$^{-2}$ cm$^2$.
2. With the increament in $P_s$, first (upto 20 kg/cm$^2$) $t_s$ decreases and then (after $P_s$ > 20 kg/cm$^2$) increases. The reason is that : low pressure cannot supply enough flow rate to make piston move fast, on the other hand, high pressure will cause overshoot and vibration of the controlled output, both of the cases will worsen the quality of the transient response.

Figure 18 shows the relation

$$t_d = f(F)/P_s = \text{const}$$

As to our expectation : the greater the pressure and the larger the initial opening area, the faster the reaction of the system.

Figure 19 shows the relation:

$$M_p = f(F)/P_s = \text{const.}$$

it turns out that for $F = (2.0 \text{ to } 3.0)10^{-2}\ cm^2$ and $P_s = (20 \text{ to } 50)\ kg/cm^2$ not only the settling time is the best (as mentioned above) but also overshoot has acceptable value $M_p < 10\%$.

A close look at Fig. 19 shows that in general, increase in $P_s$ and F will cause overshoot. For $F \leq 2.6x10^{-2}cm^2$, the higher the pressure the greater the overshoot. For $F > 2x10^{-2}\ cm^2$ there is a fall in overshoot after pressure reaches $P_s = 20\ kg/cm^2$, the fall continues when pressure reaches $P_s = 40\ kg/cm^2$ and hence fowwards, the overshoot rises again. This fact gives a hint that at some interval of supplied pressure (20 to 40 kg/cm$^2$), an increase in F can damp down the system the reason for this may link with the special feature of the bridge circuit of the servomechanism.

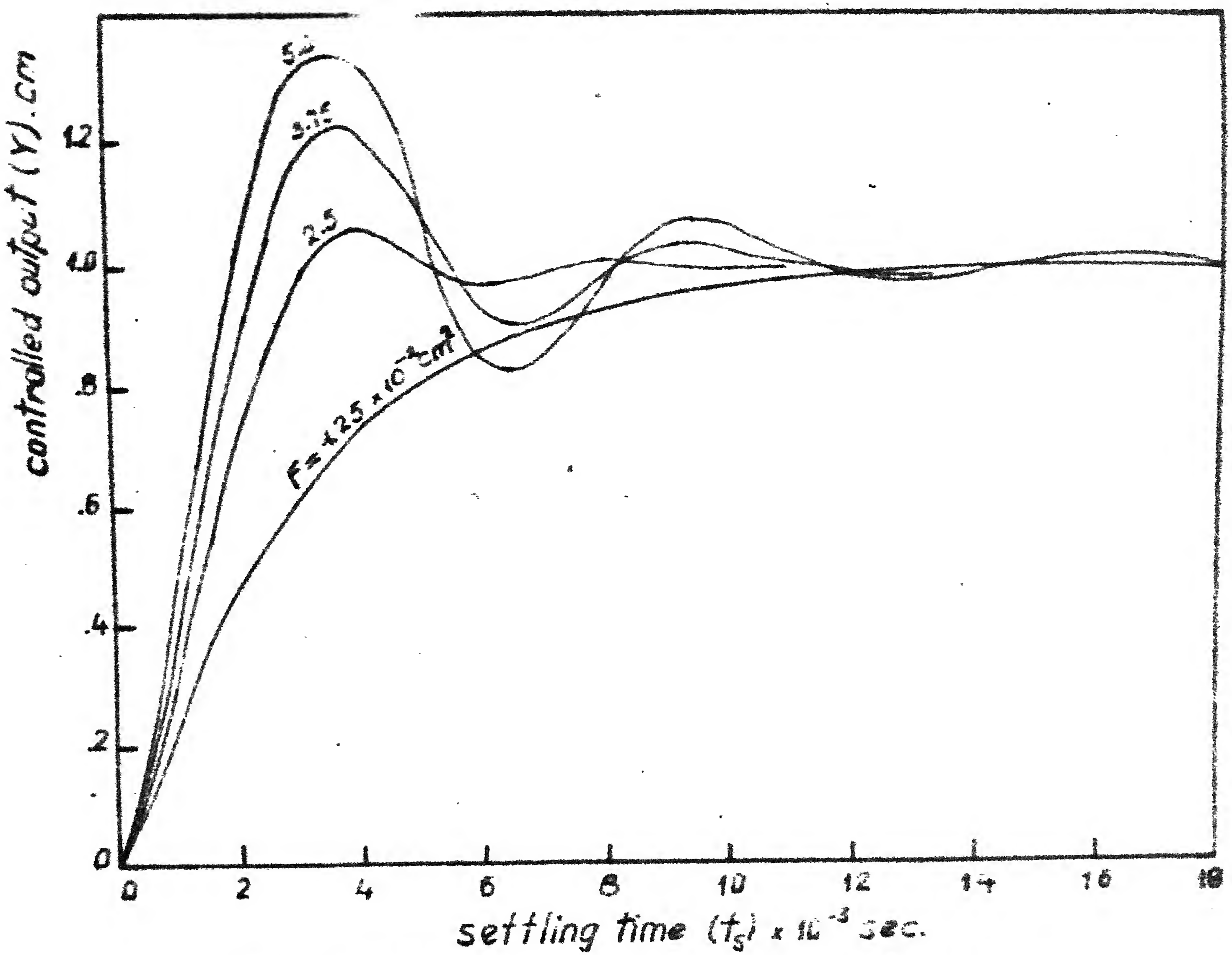


Fig. 16 Pressure ($P_s$) - opening area of the spool valve (F) - time response relation (P = 30 Kg/cm , $d_c$ = 4.5 cm)

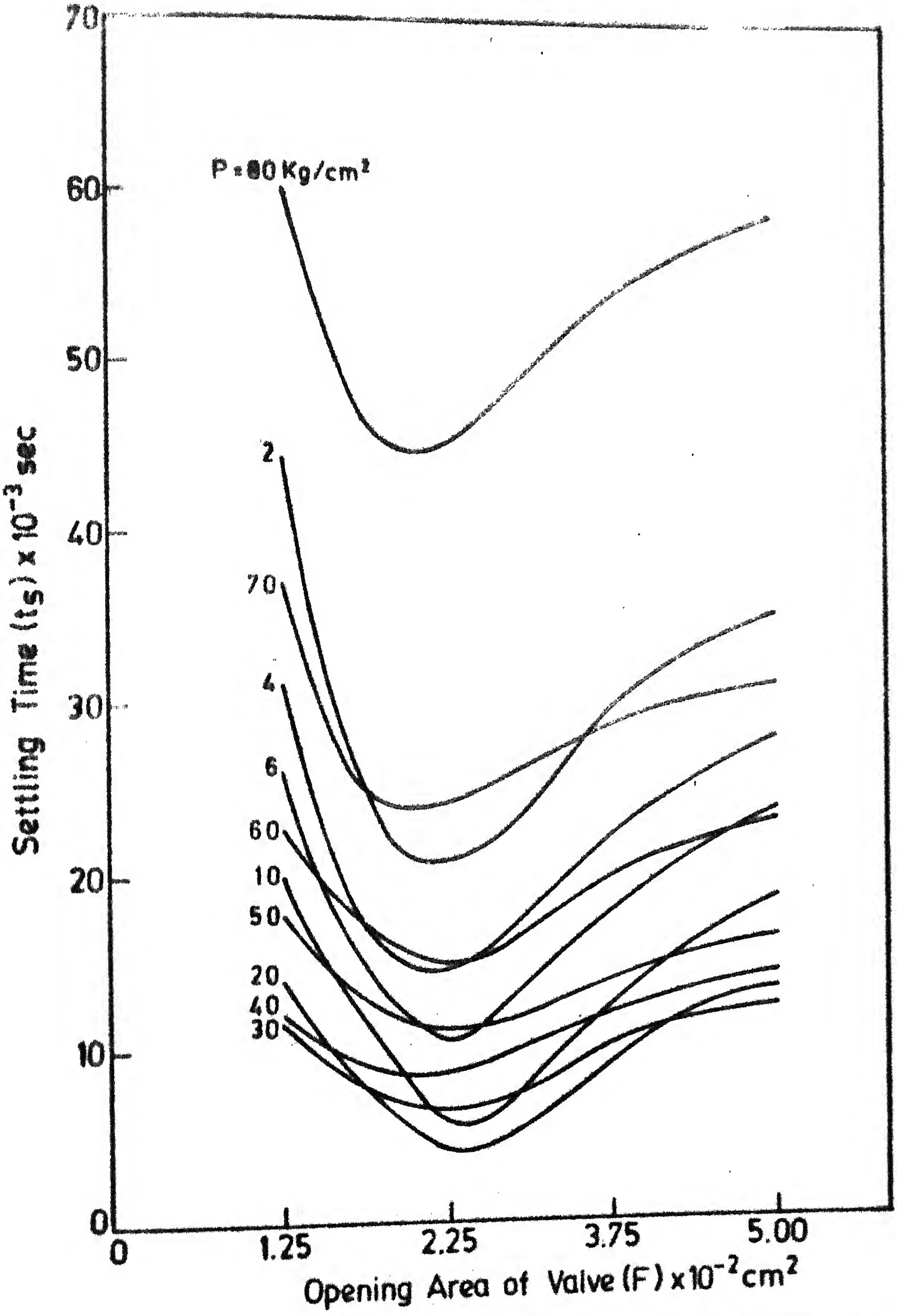


Fig. 17 Pressure ($P_s$) - opening area of valve (F) - settling time ($t_s$) relation.

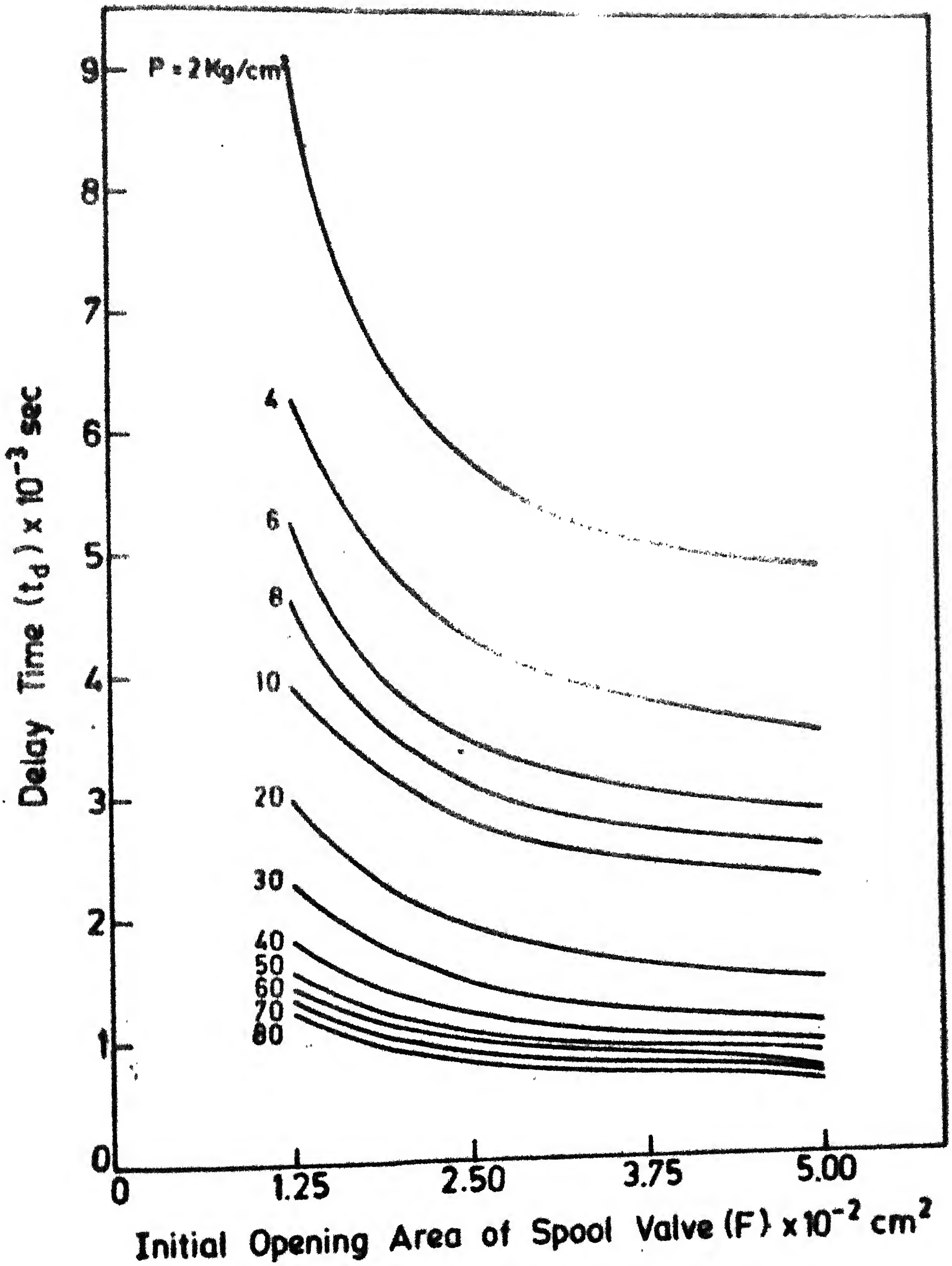


Fig. 18 $t_d$ - $P_s$ - initial opening area relation.

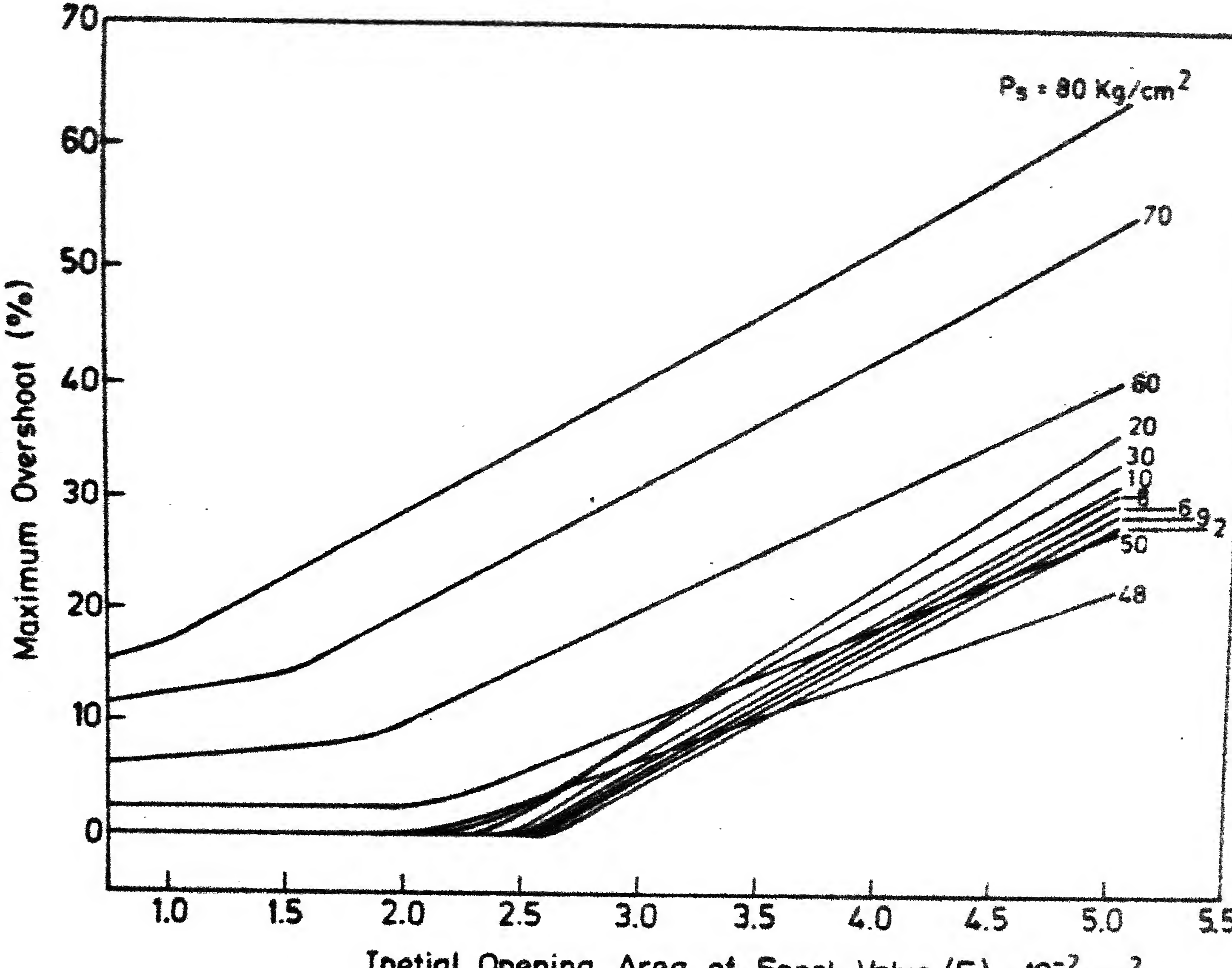


Fig. 19 Pressure ($P_s$) - maximum overshoot ($M_p$) - inetial opening area of valve (F) relation.

## 4-6. EFFECT OF INERTIA LOAD ON TRANSIENT RESPONSE

Figure 20 shows the transient response obtained when motor has diameter $d_c$ = 5cm, working under supplied pressure $P_s$ = 20 kg/cm$^2$ and inertia load varying from M = 0.05 to M = 1.0 kg.sec$^2$/cm. As to our expectation, an increase in inertia load will lead to the increase in delay time, in overshoot as well as settling time. A computation of transient response for the said set of the inertia loads acting on different motors (different $d_c$) under different pressure leads to the following:

If we denote $$R^* = \frac{P_s}{2} \times \frac{\Pi d_c^{\,2}}{4} \qquad (4\text{-}6.1)$$

and $$a = \frac{R^*}{M} \qquad (4\text{-}6.2)$$

Then it was found that : for smaller 'a' the overshoot is higher, if M = constant and for larger R* the overshoot is higher if R* = constant which means $P_s$ = const then the higher M the greater overshoot. For our design $a \geq 500$ cm/sec$^2$ there is no overshoot for $200 \leq a < 500$, the overshoot is in between (15 to 2)%; for a < 200 the overshoot is quite big for example when a = 98 cm/sec$^2$ the overshoot is 37%. To see the quality of transient response in terms of delay time we can see that if $P_s$ = const then the smaller the inertia the faster system will act; if M = Const. then the greater the pressure the faster system will act.

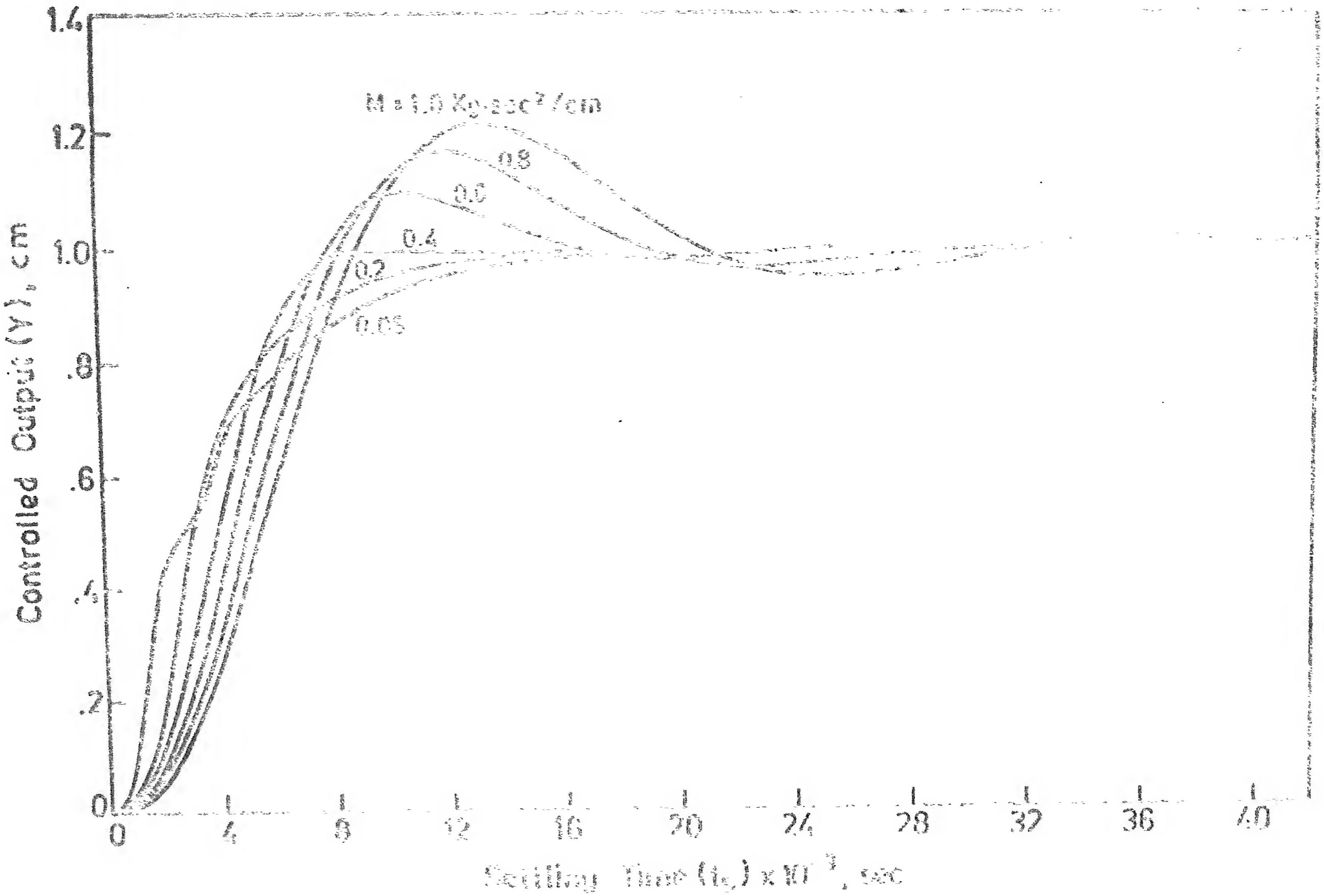


Fig. 20 Effect of inertia load.

## 4-7. EFFECT OF AIR MIXED IN THE WORKING OIL

In this section, the effect of the percentage of air mixed in the oil on transient response has been studied.

Suppose a volume, H, initially at pressure P contains a volume of air, $H_a = \varepsilon H$. The volume of oil is, therefore, $H_o = (1 - \varepsilon)H$. An increase of pressure, P, will cause a decrease of volume

$$\Delta H = \Delta H_o + \Delta H_a = \frac{H_o}{\beta e} \Delta P + \frac{H_a}{P} \Delta P \qquad (4\text{-}7.1)$$

if $\varepsilon$ is small enough $H_o = H$ and therefore

$$\Delta H = H\left(\frac{1}{\beta e} + \frac{\varepsilon}{P}\right) \Delta P \qquad (4\text{-}7.2)$$

or

$$\frac{dH}{dt} = H\left(\frac{1}{\beta_e} + \frac{\varepsilon}{P}\right)\frac{dp}{dt} \qquad (4\text{-}7.3)$$

This equationsshows that when a relative volume of air, $\varepsilon$, is mixed in the oil, the bulk modulus $\beta_e$ should be replaced by $\beta_e'$ defined by

$$\frac{1}{\beta_e'} = \frac{1}{\beta_e} + \frac{\varepsilon}{P} \qquad (4\text{-}7.4)$$

The effect of various percentage of air mixed in oil under the range of possible supplied pressure is shown in Figure 21.

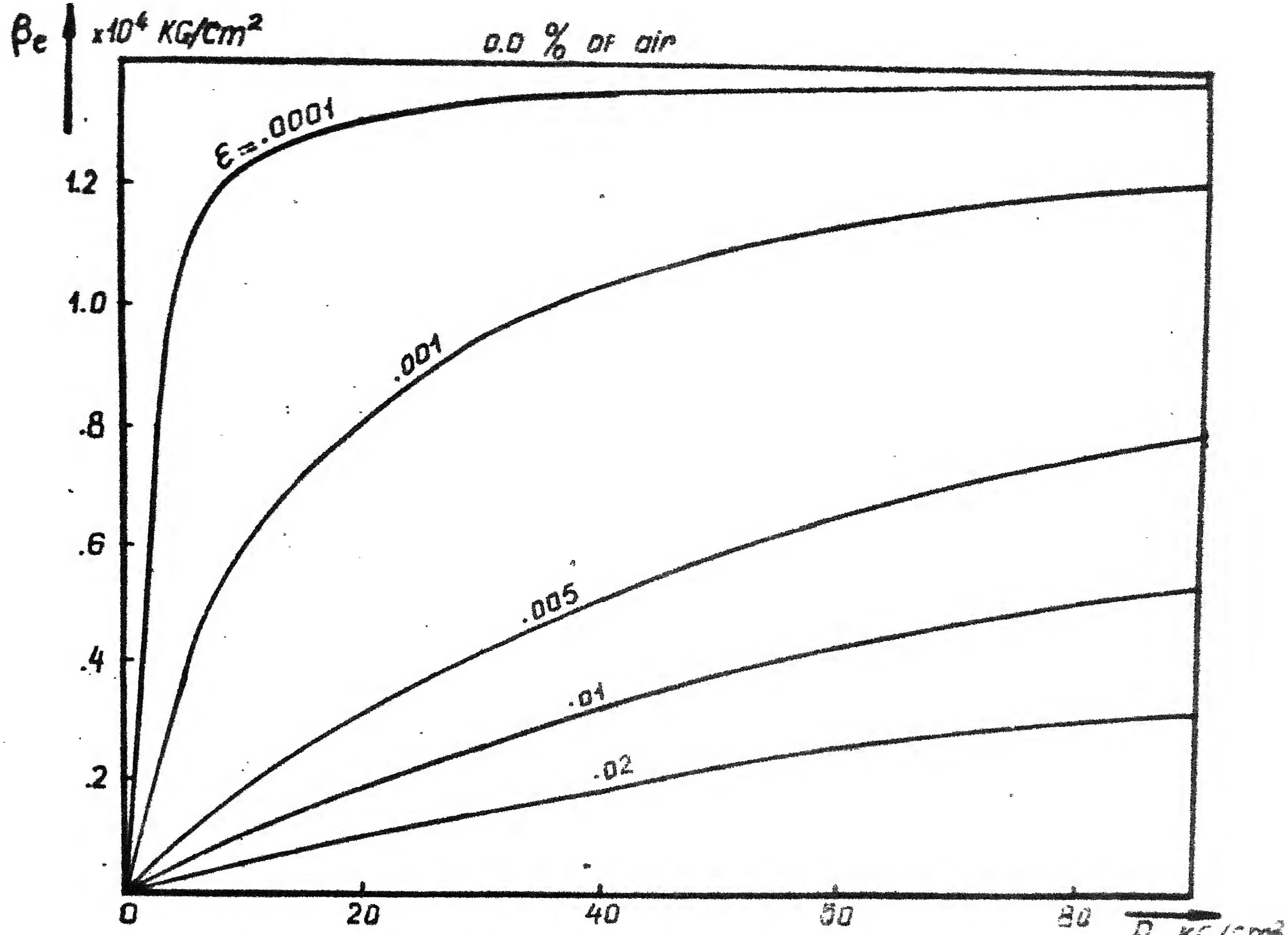


**Fig. 21** Effect of air on bulk modulus of oil on transient response.

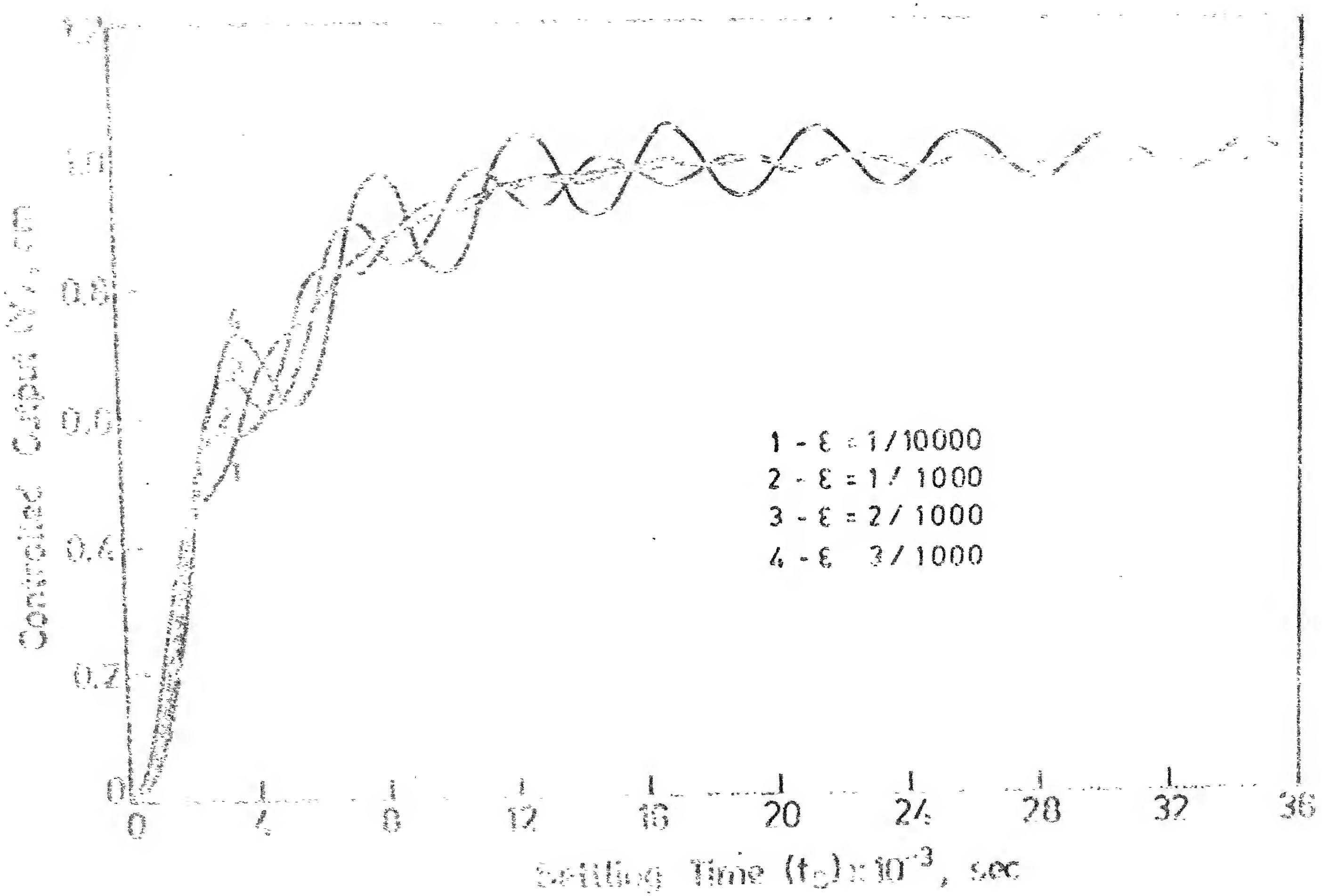


Fig. 22 Effect of air mixed in the oil.

# CHAPTER V

## ON TRANSIENT RESPONSE OF THE SYSTEM WORKING IN CONSTANT FLOW-RATE REGIME

As stated earlier, the advantage of the chosen model is that when the initial opening area of the spool-valve is big enough, the system can work in constant flow-rate regime. In this case, the power unit supplies a constant flow rate to the system and supplied pressure is determined by the load, acting on the driven element of the motor.

One of the possible half bridge circuit is shown in Fig. 1a. When $R_1 = R_2$ and the constant flow-rate $Q_1 = Q_2 = Q_{s0} = C_d.W.X_o.B.\sqrt{P_{so}}$ is supplied to the motor chambers from a flow divider or from two identical pumps driven by the same shaft [9].

It was known [10] that hydraulic servomechanism, working in constant flow rate regime has many advantages such as: high efficiency, low temperature of working oil, longivity of service, high sensitivity to the input and the main disadvantage is that, the system has very low stiffness, which results in low position accuracy of the controlled output. This may be reason, why hydraulic servomechanims working in

flow rate regime have found their application mostly in transportation machines like : tractors, heavy trucks and other farming machines, where the energy requirement is of prime important.

Under the same conditions and assumptions, following the same steps used in Chapter II, we can get the same type of transfer function depicted by Eqns. (2-69) and (2-65). The block diagram similar to that shown in Figs. 4a,b,c, also can be obtained, the only difference is that the constants have the following values:

$$T_1 = \frac{E.V.M}{2A^2}\ ; \qquad T_2 = \frac{M.C_d.W.B.X_o}{4.\sqrt{P_{so}}.A^2}$$
$$K_c = \frac{C_d.W.B.\sqrt{P_{so}}}{A}; \quad T_r = \frac{2.E.V.\sqrt{P_{so}}}{C_d.W.B.X_o} \qquad (5\text{-}1)$$
$$K_r = \frac{C_d.W.B.\ \sqrt{P_{so}}}{4.A}\ ; \quad f_r = \frac{\Delta R}{A.P_{so}}$$

where $P_{so}$ - supplied pressure to the system when load is absent.

Applying Routh's criteria to check the system stability leads to the following condition

$$P_{so} < \frac{X_o}{K_1.E.h} \qquad (5\text{-}2)$$

Comparing the above inequality with that shown by Eqn. (3.8) one can see that the range of $P_{so}$ is a half of that of $P_s$. This observation means : in case only inertia load is acting, if $P_s = P_{so}$ then constant flow rate regime is closer to the border of stability than constant supplied pressure regime In other words, the degree of stability of the former is less than that of the latter.

The qual power input condition of the system working in the two regimes leads to the following:

$$P_{so} = \sqrt[3]{\frac{r^2}{2}}\, P_s \qquad (5\text{-}3$$

where r is a constant coefficient which shows what part of the supplied flow rate should be continuously relieved through the relief valve so that supplied pressure $P_s$ is maintained constant theoretically $r \geq 1$.

The obtained results for the case when $P_s = 20 kg/cm^2$ with r varying from 1.0 to 1.4 are shown in Fig. 23.

Similarly, the equal pump flow rate condition leads to the following

$$P_{so} = \frac{r^2}{2} P_s \qquad (5\text{-}4)$$

The obtained results for the case when $P_s = 20\ kg/cm^2$ with a varying from 1.0 to 1.4 are shown in Fig. 24.

Figure 25 shows the transient curves of the system working in constant flow rate regime under equal effective flow rate conditions, i.e.:

$$P_{so} = \frac{1}{2} P_s \qquad (5\text{-}5)$$

The curves obtained for the case when $P_s$ = 10, 20, 30 and 40 $kg/cm^2$ accordingly $P_{so}$ = 5, 10, 15 and 20 $kg/cm^2$.

As to our expectation, in any case, the quality of transient response of the system working in constant flow rate regime is lower than that of the system working in constant supplied pressure regime.

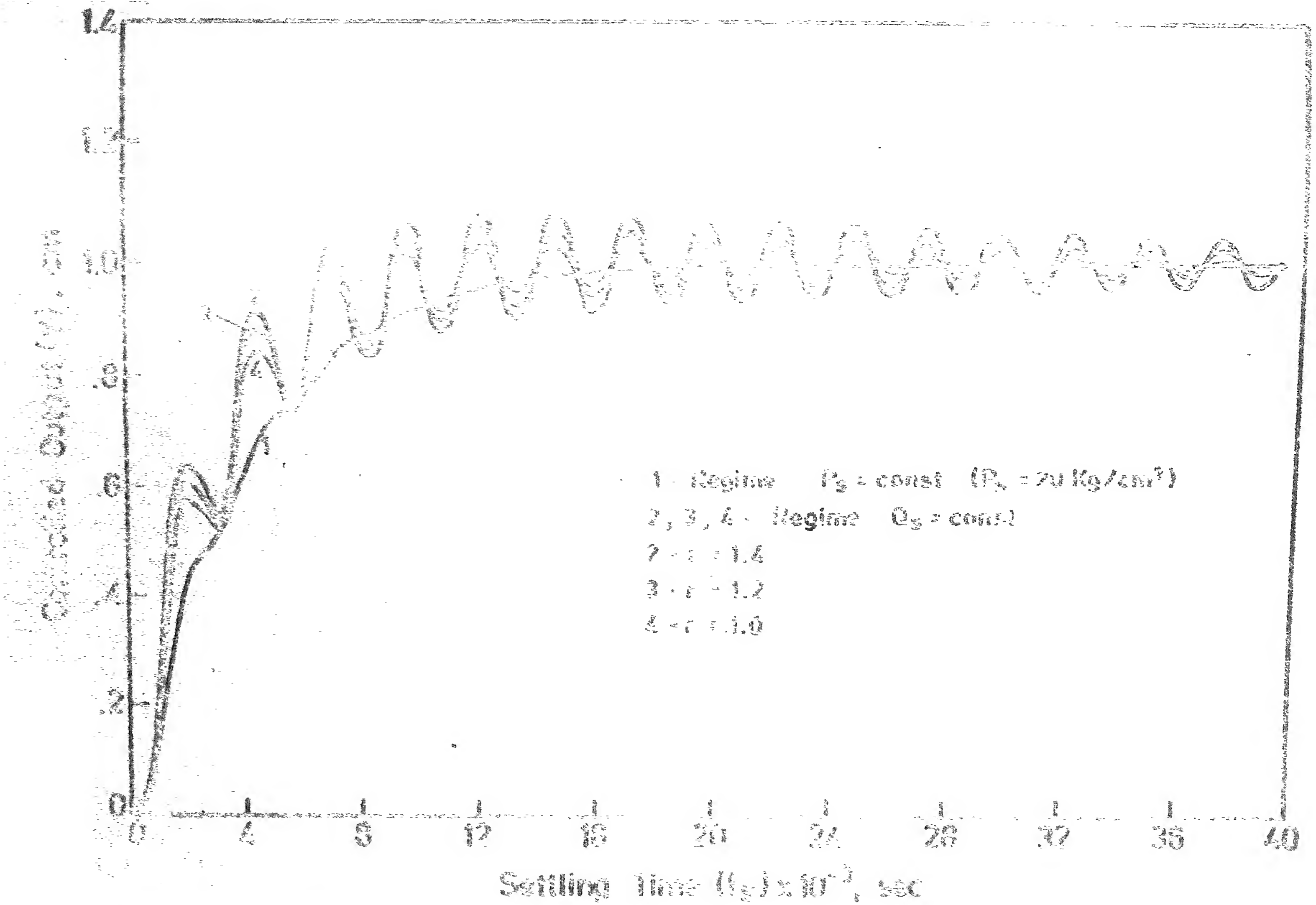


Fig. 23 Transient response of two regimes working under equal input power condition ($P_s$ = 20 Kg/cm , r ≠ const.)

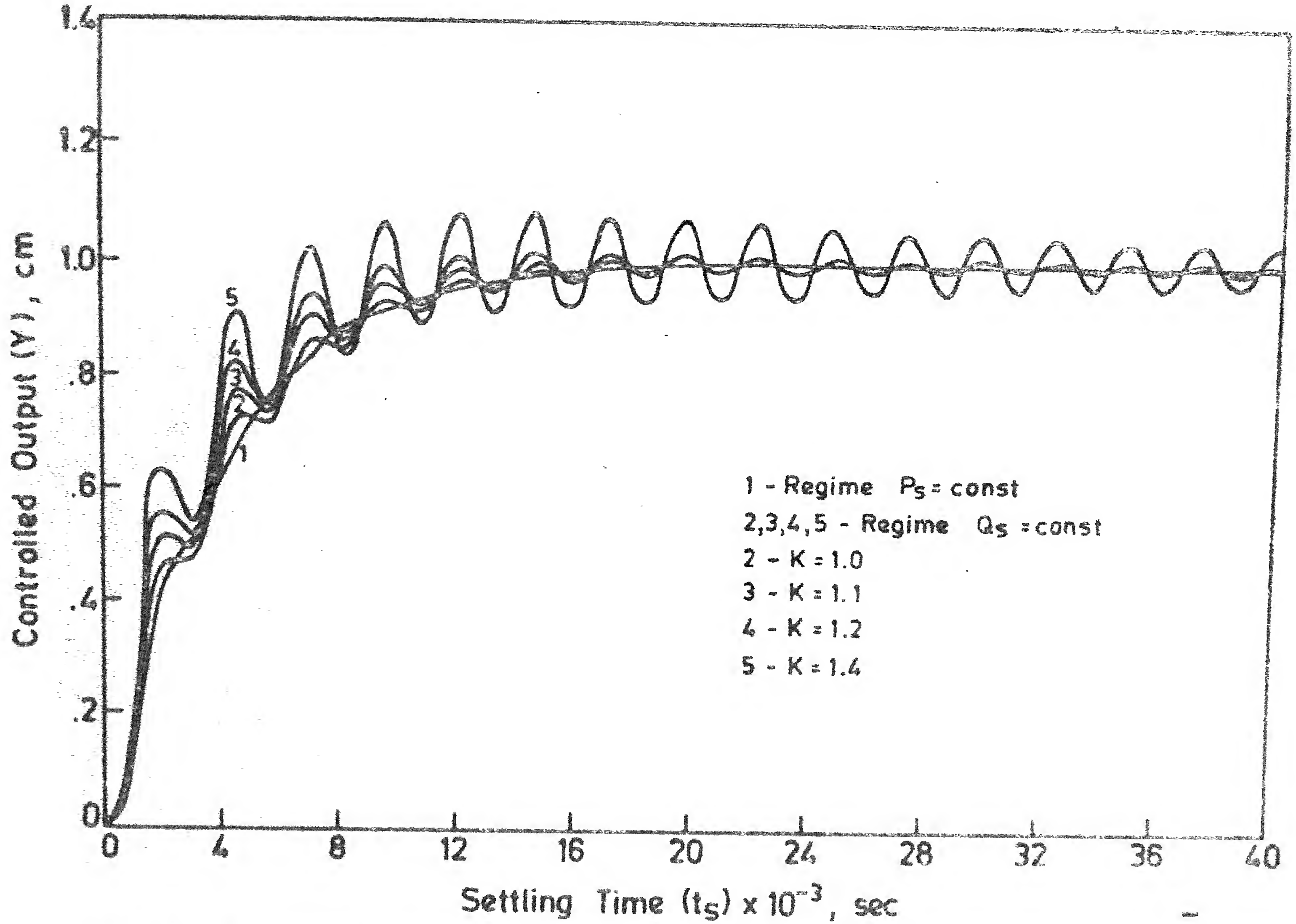


Fig. 24 Transient response of two regimes working under equal pump-flow-rate condition (P = 20 Kg/cm , K ≠ const.)

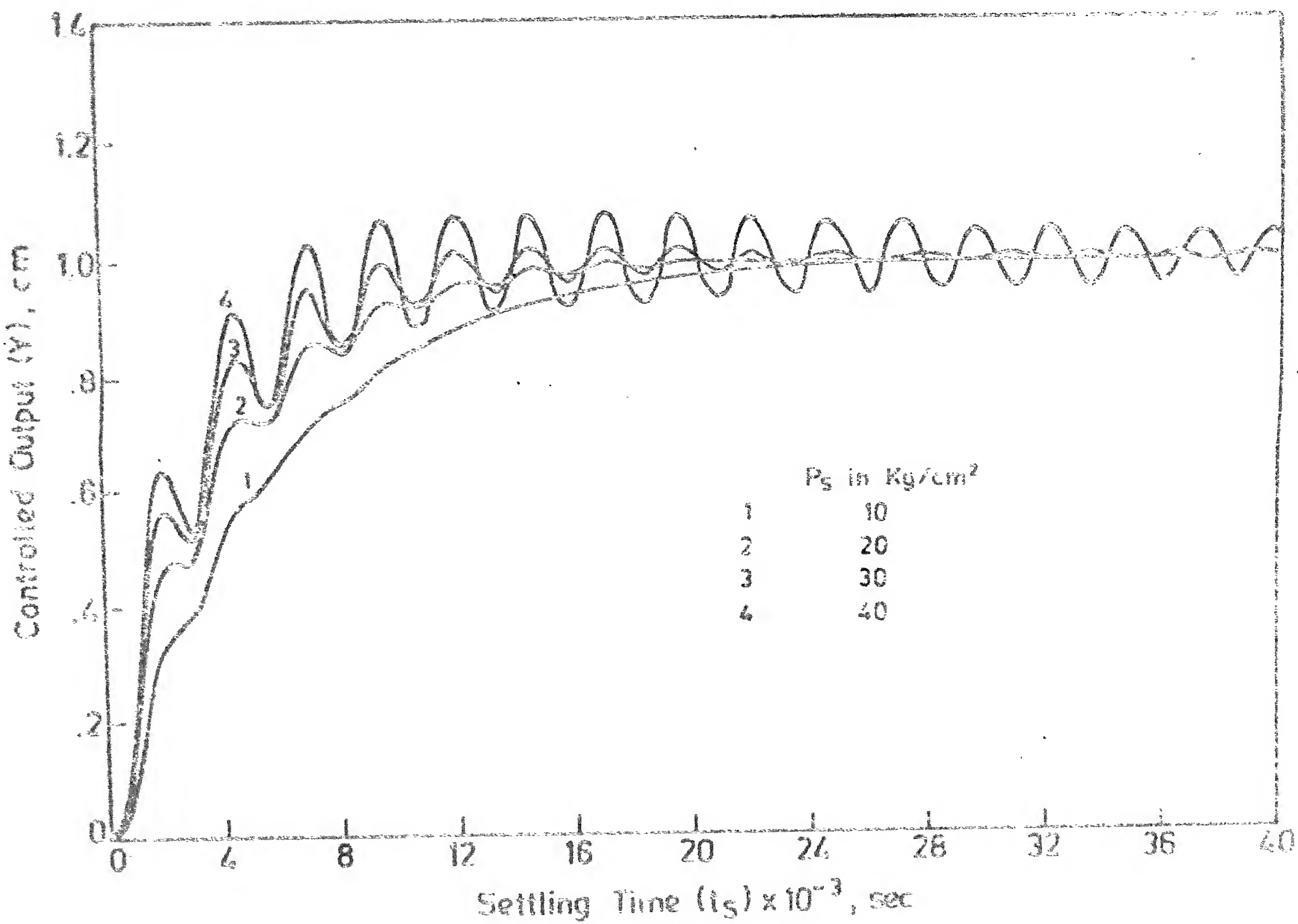


Fig. 25 Transient response of the system in regime $Q_s$ = const working under equal effective-flow-rate condition.

# CHAPTER VI

## CONCLUSION and SUGGESTION

Having discussed all the obtained results given in previous chapters, this chapter is merely to summarize the most important points.

(1) An acceptable mathematical model of the hydraulic servomechanism controlled by under lap 4-way spool valve has been developed.

(2) From stability point of view, by neglecting leakages in spool valve and in motor chambers, a safety factor is automatically obtained. This is obvious because leakages play a damping role, they serve [8] as one of the powerful method to stabilize the system.

(3) Using Routh's criteria, the relation $P_s(K_1)$ for different possible non-dimensional factor, where the system stability is guaranteed has been obtained. The graph shown in Fig. 5 can be used as a base to select $P_s$ in accordance with $K_1$.

(4) A criteria to design a linear motor (Fig. 12), working under given pressure ($P_s$) and controlled by a standard under-lap spool valve, having the desired quality of transient response has been obtained.

(5) Graph shown in Fig. 17 allows us to select or to design a suitable under lap spool valve for a given linear motor, working under known supplied pressure ($P_s$) which has the desired quality of Transient Response.

(6) Under the same conditions, the chosen model while working in constant supplied pressure regime has higher degree of stability than the case when working in constant flow-rate regime.

(7) The presence of air in the oil slows down the reaction of the system and lengthens the settling time due to vibration of the controlled output.

It should be noted that the above main points together with the remarks made in previous chapters make this study applicable. There is no doubt that the obtained results can be directly used to design or to predict dynamic performance of hydraulic servomechanims of the chosen type.

In view of the simplicity of the proposed method, it is suggested that other models which are widely used in practice should be given the same treatment so as to find out the general conclusions which are applicable for all hydraulic servomechanisms.

## REFERENCES


1. Lee, S.Y., .... "Contribution to Hydraulic Control", ASME Trans., Vol. 74, No. 6, pp. 1005-1011, 1952.

2. William, S.B., .... "Hydraulic Components", McGraw-Hill, 1956.

3. G.F. Kelley, ...., "Designs of Hydraulic Machines", Trans. AIEE, 61, 502-570, 1960.

4. Blackburn, J.F., "Transition Flow Forces and Valve Instability", ASME Trans., Vol. 74, No. 6, pp. 1013-1016, 1952.

5. Minorsky, N., "Directional Stability of Automatically Steered Bodies", 1922, J. Amer. Soc. of Nav. Engg., Vol. 34, pp. 280.

6. Routh, E.J., "Advanced Rigid Dynamics", MacMillon, London, 1984.

7. Hurwitz, A., "On the Condition Under which an Equation has Only Roots With a Negative Real Part", Math. Ann., 46, 73, 1973.

8. M. Guillon, "Hydraulic Servo Systems Analysis and Design", London Butter Worths, 1969.

9. Khaimovich, E.M., "Hydraulic Control of Machine Tools", McMillon, London, 1965.

10. Tumarkin, M.L., "Hydraulic Follow-up Systems" Machine, Moscow, 1966.

11. Handha, E.C., "A Study on Hydraulic Servomechanism for Position Control", M.Tech. Thesis, IIT Kanpur, 1968.

12. Bode, H.W., "Network Analysis and Feedback Amplifier Design", D. Van Nostrand Company Inc., Princeton, N.J., 1945.

13. Nyquist, H., "Regeneration Theory", Bell System Tech. J., 11, pp. 126-147, 1932.

14. Evan, W.R., "Graphical Analysis of a Control Systems", Trans. AIEE, 67, 547-551, 1948.

15. Nagrath, Gopal, "Control Systems Engineering", Wiley Eastern, Second Edition, 1983.

16. NAG Program Manual, Computer Centre, Indian Institute of Technology, Kanpur.